1. $\nabla f = \begin{bmatrix} 2x-2 \\ 2y+2 \end{bmatrix}$. $\nabla g = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$.

The critical points inside the region are where $2x-2=0 \Rightarrow x=1$, and $2y+2=0 \Rightarrow y=-1$, i.e. $\boxed{(x,y)=(1,-1)}$.

To find critical points on the boundary, we may use Lagrange multipliers. We obtain:

$$\left.\begin{array}{l} \text{(i) } 2x-2 = \lambda(2x) \\ \text{(ii) } 2y+2 = \lambda(2y) \end{array}\right\}$$

$$\text{(iii) } x^2+y^2=4$$

Adding (i) and (ii) gives $2x+2y=\lambda(2x+2y)$, which implies that either $\lambda=0$ or $y=-x$.

$\lambda=0$ gives $x=1$, $y=-1$, not satisfying (iii), so $y=-x$, giving $2x^2=4 \Rightarrow \boxed{x=\sqrt{2}\cancel{/2},\ y=-\sqrt{2}\cancel{/2}}$ or $\boxed{x=-\sqrt{2}\cancel{/2},\ y=\sqrt{2}\cancel{/2}}$

Thus, the candidates for optima are:

x	y	$f(x,y)$
1	-1	0
$\sqrt{2}\cancel{/2}$	$-\sqrt{2}\cancel{/2}$	$\approx$ ~~.1716~~ .3431
$-\sqrt{2}\cancel{/2}$	$\sqrt{2}\cancel{/2}$	$\approx$ ~~5.8284~~ 11.6569

Thus, the minimum ~~in~~ the region is 0, and it occurs at $x=1$, $y=-1$, while the maximum over the region is about ~~5.8284~~ 11.6569, and it occurs at $x=-\sqrt{2}\cancel{/2}$, $y=\sqrt{2}\cancel{/2}$.

2 $\nabla f = \begin{bmatrix} \cancel{3}x^2 - y^2 \\ -2xy + 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ where $2y(-x+1) = 0$
$\Rightarrow x = 1$ or $y = 0$.

When $x = 1$, $y = 1$ or $y = -1$. When $x = 0$, $y = 0$.

$\dfrac{\partial^2 f}{\partial x^2} = 2x$, $\dfrac{\partial^2 f}{\partial y^2} = -2x + 2$. $\dfrac{\partial^2 f}{\partial y \partial x} = -2y$,

so the discriminant is $D(x,y) = \dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial y \partial x}\right)^2$

$= (2x)(-2x+2) - (-2y)^2$

We thus have the following critical points:

x	y	D	f_{xx}	type of point
1	1	-4		saddle point
1	-1	-4		saddle point
0	0	0		~~no~~ saddle point

The discriminant is 0 at $x = y = 0$. However, by looking at what happens in the cross-sections with $x = 0$ and with $y = 0$, we see that $(0,0)$ corresponds to a saddle point.