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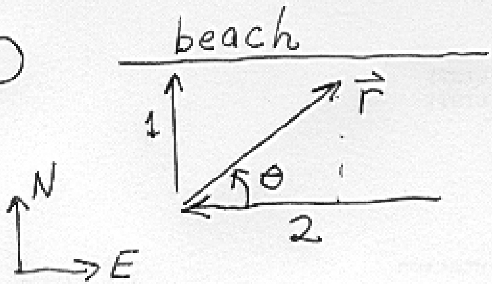

If $\vec{r}$ is the person's velocity vector, then the current vector is $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$, the desired resultant vector is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and we must have

$$\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \vec{r} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \text{ giving } \vec{r} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

The person's speed would need to be

$$\|\vec{r}\| = \sqrt{2^2+1^2} = \sqrt{5} \approx 2.2 \text{ miles per hour.}$$

The person's angle with respect to east will obey: $\theta = \arctan\left(\frac{1}{2}\right) \approx .464 \text{ radians} \approx 27^\circ$

2 Let $U(t) = T(x(t), y(t), z(t))$ be the temperature at
a time T. Then $x(60) = 3600$, $y(60) = 3600$, $z(60) \approx 465$,
and $U(1\text{ min.}) = U(60\text{sec}) \approx 25 - .0001(3600) - .0002(3600) - .005(465)$
≈ 21.596 ~~21.596~~ degrees centigrade

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b $U(t) = 25 - .0001t^2 - .0002t^2 - .005t^{3/2}$
$= 25 - .0003t^2 - .005t^{3/2}$

(i) so $\dfrac{dU}{dt} = -.0006\,t - .0075\sqrt{t}$,

so $\left.\dfrac{dU}{dt}\right|_{t=60} \approx -.0006(60) - .0075\sqrt{60} \approx -.094$ degrees per second

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(ii) $\dfrac{dU}{dt} = \dfrac{dT}{dx}x' + \dfrac{dT}{dy}y' + \dfrac{dT}{dz}z'$.

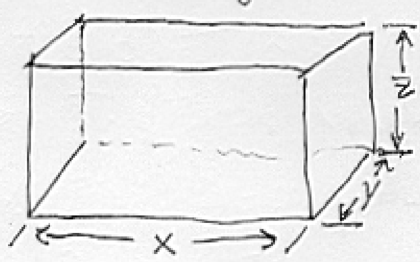
$\dfrac{dT}{dx} = -.0001$; $\dfrac{dT}{dy} = -.0002$; $\dfrac{dT}{dz} = -.005$

$x' = 2t$; $y' = 2t$; $z' = \frac{3}{2}\sqrt{t}$.

$x'(60) = 120$; $y'(60) = 120$; $z'(60) \approx 11.6$

Thus, $\dfrac{dU}{dt} \approx (-.0001)(120) + (-.0002)(120) + (-.005)(11.6)$
$\approx -.094$ degrees per second

3



We have the cost is

$$C(x,y,z) = (10+1)xy + (4)(2)xz + (2)(2)(yz)$$

subject to the constraint

$$xyz = 10.$$

The problem is thus

minimize $C(x,y,z) = 11xy + 8xz + 4yz$

subject to $g(x,y,z) = xyz = 10.$

Using Lagrange multipliers, we obtain:

$$\nabla C + \lambda \nabla g = \begin{bmatrix} 11y + 8z \\ 11x + 4z \\ 8x + 4y \end{bmatrix} + \lambda \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It seems more complicated to solve this than simply plugging the constraint into the objective and reducing it to a function of 2 variables, so we'll do that: $z = 10/(xy)$,

so $\tilde{C}(x,y) = C(x, y, 10/(xy)) = 11xy + \frac{80}{y} + \frac{40}{x}$.

$$\frac{\partial \tilde{C}}{\partial x} = 11y - \frac{40}{x^2} = 0; \quad \frac{\partial \tilde{C}}{\partial y} = 11x - \frac{80}{y^2} = 0$$

This gives $y = \frac{40}{11}\frac{1}{x^2}$, whence $11x - \frac{80}{\left(\frac{40}{11}\cdot\frac{1}{x^2}\right)^2} = 0.$

Simplifying the latter gives $11x - \frac{121x^4}{20} = 0,$

or $11x\left(1 - \frac{11}{20}x^3\right) = 0$, whence $x = \left(\frac{20}{11}\right)^{1/3} \approx \boxed{1.22 \approx x}$

$$y \approx \frac{40}{11}\cdot\frac{1}{1.22^2} \approx \boxed{2.44 \approx y}; \quad z \approx \frac{10}{(1.22)(2.44)} \approx \boxed{3.35 \approx z}$$

The cost of the box will then be

$$11(1.22)(2.44) + 8(1.22)(3.35) + 4(2.44)(3.35) \approx \boxed{\$98.14}$$

4

$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \iint_{R} \operatorname{curl}(\vec{F})\, dA$$ ← Green's Theorem

$$\operatorname{curl}(\vec{F}) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \cancel{0}\,1-0 = \cancel{0}\,1.$$

Thus, $$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \iint_{R} 1\cdot dA = 1\cdot[\text{Area of unit circle}] = \boxed{\pi}.$$

5 $\displaystyle\iint_R \vec{F}\cdot\vec{n}\,dA = \iint_{s\,t} \left(\frac{\partial \vec{r}}{\partial s}\times\frac{\partial \vec{r}}{\partial t}\right)\cdot\vec{F}\,ds\,dt.$

We can parametrize the paraboloid by

~~R(t) =~~ $x(s,t) = s$

$y(s,t) = t$

$z(s,t) = s^2+t^2, \quad -1\le s\le 1,\ -1\le t\le 1.$

Thus, $\vec{r}(s,t) = \begin{bmatrix} s \\ t \\ s^2+t^2 \end{bmatrix}$, so $\dfrac{\partial \vec{r}}{\partial s} = \begin{bmatrix} 1 \\ 0 \\ 2s \end{bmatrix}$, $\dfrac{\partial \vec{r}}{\partial t} = \begin{bmatrix} 0 \\ 1 \\ 2t \end{bmatrix}$,

and $\vec{F} = \begin{bmatrix} s \\ t \\ s^2+t^2 \end{bmatrix}$. Therefore, to within a factor of -1,

$$\vec{F}\cdot\vec{n}\,dA = \pm \begin{vmatrix} 1 & 0 & 2s \\ 0 & 1 & 2t \\ s & t & s^2+t^2 \end{vmatrix} ds\,dt$$

$$= \pm\left(1\begin{vmatrix} 1 & 2t \\ t & s^2+t^2 \end{vmatrix} + 2s\begin{vmatrix} 0 & 1 \\ s & t \end{vmatrix}\right) = \pm(s^2+t^2-2t^2-2s^2) = \pm(-s^2-t^2).$$

Since the flux must be positive, we take s^2+t^2, and the total flux is: $\displaystyle\int_{s=-1}^{1}\int_{t=-1}^{1}(s^2+t^2)\,dt\,ds$

$$= \int_{s=-1}^{1} s^2t+\frac{t^3}{3}\Big|_{t=-1}^{1} ds = \int_{s=-1}^{1} 2s^2+\frac{2}{3}\,ds = \left[\frac{2}{3}s^3+\frac{2}{3}s\right]\Big|_{s=-1}^{1}$$

$$= \boxed{\frac{8}{3}}$$

6 The divergence theorem states that:

$$\iint_{S} \vec{F} \circ \vec{n}\, dA = \iiint_{V} \operatorname{div}(\vec{F})\, dV, \text{ where } \operatorname{div}(\vec{F}) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}$$

Since $\vec{F}(x,y,z) = \begin{bmatrix} y^2/2 \\ -xy+y \\ xz-yz \end{bmatrix}$,

$$\operatorname{div}(\vec{F}) = 0 + (-x+1) + (x-y) = 1-y.$$

Therefore, $\displaystyle\iint_{S} \vec{F} \circ \vec{n}\, dA = \iiint_{V} (1-y)\, dV$

$$= \iiint_{V} (1)\, dV - \iiint_{V} y\, dV = 1\begin{bmatrix}\text{volume of} \\ \text{unit ball}\end{bmatrix} - 0$$

$$= 4/3\pi,$$

where the second triple integral is 0 from symmetry.