(1) $\iint_{\mathcal{S}} \vec{F} \circ \vec{n}\, dS = \iiint_{\mathcal{V}} \operatorname{div}(\vec{F})\, dV.$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=0}^{1} \int_{\varphi=0}^{\pi} (x^2+y^2+z^2)\rho^2 \sin(\varphi)\, d\varphi\, d\rho\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \left\{ \int_{\rho=0}^{1} \int_{\varphi=0}^{\pi} \rho^4 \sin(\varphi)\, d\varphi\, d\rho \right\} d\theta = \cdots$$

$$= \left\{ \int_{\theta=0}^{2\pi} d\theta \right\} \left\{ \int_{\varphi=0}^{\pi} \sin\varphi\, d\varphi \right\} \left\{ \int_{\rho=0}^{1} \rho^4\, d\rho \right\}$$

$$= (2\pi)(2)\left(\frac{1}{5}\right) = \boxed{\frac{4}{5}\pi}$$

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(2) The interior of the square is given by $0 \le x \le 1, 0 \le y \le 1$, while $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 1 - 0 = 1$. Thus,

$$\oint_{\mathcal{C}} \vec{F} \circ d\vec{r} = \int_{x=0}^{1} \int_{y=0}^{1} 1\, dy\, dx = \boxed{1}.$$

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(3) $\oint_{\mathcal{C}} \vec{F} \circ d\vec{r} = \iint_{\mathcal{S}} \operatorname{curl}(\vec{F}) \circ \vec{n}\, dS$

$$\operatorname{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & y \end{vmatrix} = \vec{i}(1) - \vec{j}(0) + \vec{k}(0) = \langle 1, 0, 0 \rangle.$$

For $\vec{n}\, dS$, $\vec{r} = \langle 1, y, z \rangle$, $0 \le y \le 1, 0 \le z \le 1$,

so $\frac{\partial \vec{r}}{\partial y} = \langle 0, 1, 0 \rangle$ and $\frac{\partial \vec{r}}{\partial z} = \langle 0, 0, 1 \rangle$,

and . . .

$$\vec{n}\,dS = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} dx\,dy = \vec{i} = \langle 1,0,0\rangle.$$

Thus, $\displaystyle\iint_S \mathrm{curl}(\vec{F}) \circ \vec{n}\,dS = \int_{z=0}^{1}\int_{y=0}^{1} \langle 1,0,0\rangle \circ \langle 1,0,0\rangle\,dy\,dz$

$$= \int_{z=0}^{1}\int_{y=0}^{1} 1\,dy\,dz = \boxed{1}$$