Math. 302-04
Fall, 2017
R. B. Kearfott

Final Examination
Monday, December 4, 2017
2:00PM to 4:30PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 14 points, and 2 points are free.

1. Determine an equation for the plane containing the point $(1,0,0)$ and perpendicular to the line through $(1,2,3)$ and $(4,5,6)$.

2. Derive an equation for the tangent plane to $f(x,y) = x\cos(xy)$ at $x = 1$, $y = \pi$.

3. Find the absolute maximum and absolute minimum of
$$f(x,y) = \frac{1}{3}x^3 - xy^2 + \frac{1}{2}y^2 + 1$$
over the square $-1 \le x \le 1$, $-1 \le y \le 1$.

4. Compute
$$\iiint_{\mathcal{V}} x^2 + y^2 \; dV,$$
where $\mathcal{V}$ is the portion of the interior of the cylinder $x^2 + y^2 \le 1$ between $z = 0$ and $z = 2$.

5. Compute the surface integral
$$\iint_{\mathcal{S}} \langle x, y, z\rangle \cdot \vec{n} \; dS,$$
where $\mathcal{S}$ is the surface parametrized by
$$x(u,v) = \sin(u)\cos(v), \quad y(u,v) = \sin(u)\sin(v), \quad z(u,v) = \cos(u),$$
$0 \le u \le \pi$, $0 \le v \le 2\pi$.
(*There is more than one way of doing this one. Choose the easiest way.*)

6. Consider the vector field $\vec{F}(x,y) = \langle x^2 - y^2, -2xy + y\rangle$.
   (a) Show that $\vec{F}$ is conservative.
   (b) Compute $\displaystyle\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\mathcal{C}$ is any path with starting point $(0,0)$ and ending point $(1,1)$.

7. Use Stokes' theorem to compute the circulation of $\vec{F}(x,y,z) = \langle x, y, z\rangle$ around the unit circle $x^2 + y^2 = 1$, oriented counterclockwise in the (x,y) plane.