(1) $\tilde{b} = \frac{1}{\|\vec{b}\|}\vec{b} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$. $\vec{v} = (\vec{u} \circ \tilde{b})\tilde{b} = (\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}})\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$= \boxed{\langle \frac{2}{3}, \frac{2}{3}, \frac{2}{3} \rangle = \vec{v}}$.

$\vec{w} = \vec{u} - \vec{v} = \langle -1, 1, 2 \rangle - \langle \frac{2}{3}, \frac{2}{3}, \frac{2}{3} \rangle = \boxed{\langle \frac{-5}{3}, \frac{1}{3}, \frac{4}{3} \rangle = \vec{w}}$

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(2) Vectors in the plane are $\vec{v} = \langle 1,0,1 \rangle - \langle 1,-1,1 \rangle = \langle 0,-1,0 \rangle$ and $\vec{w} = \langle 1,2,3 \rangle - \langle 1,-1,1 \rangle = \langle 0,3,2 \rangle$.

a normal to the plane is thus

$$\langle 0,-1,0 \rangle \times \langle 0,3,2 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 0 \\ 0 & 3 & 2 \end{vmatrix} = -2\vec{i} + 0\vec{j} + 0\vec{k},$$

so an equation for the plane is

$$(\langle x,y,z \rangle - \langle 1,-1,1 \rangle) \circ (\langle -2,0,0 \rangle) = 0, \text{ i.e. } \boxed{x = 1}.$$

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(3) $\vec{r}\,'(t) = \langle \frac{1}{t}, 1, e^{t-1} \rangle$, so $\vec{r}\,'(1) = \langle 1,1,1 \rangle$.

$\vec{r}(1) = \langle 0,1,1 \rangle$, so the tangent line has

$\vec{l}(t) = \langle 1,1,1 \rangle + t\langle 0,1,1 \rangle$, i.e $\boxed{x(t) = \cancel{1+}t,\ y(t) = 1+t,\ z(t) = 1+t.}$

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(4) $\nabla f = \langle 1+y, x+2y, -2z \rangle$, so $\cancel{\nabla f(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}) = \langle 1, \frac{1}{\sqrt{2}}, \sqrt{2} \rangle}$,

~~so~~ $\cancel{D_{\vec{u}} f}$ so $\nabla f(1,0,2) = \langle 1,1,-4 \rangle$,

so $D_{\vec{u}} f(1,0,2) = \langle 1,1,-4 \rangle \circ \langle \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \rangle$

$= \frac{1}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \boxed{\frac{5}{\sqrt{2}}}$

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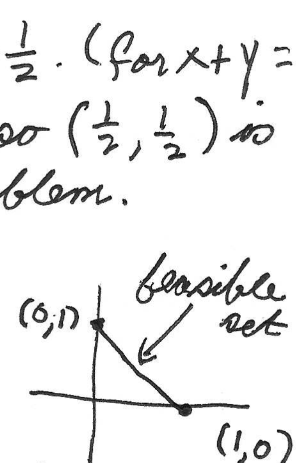
5 $\nabla f = \langle 2x, 2y \rangle$ and $\nabla g = \langle 1, 1 \rangle$.

$\nabla f = \lambda \nabla g \Rightarrow 2x = \lambda,\ 2y = \lambda \Rightarrow x = y = \frac{1}{2}$. (for $x+y=1$)

This satisfies the constraint $g = 1$, so $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a critical point of the constrained problem.

We also need to check $(0,1)$ and $(1,0)$.



We have

(x,y)	$f(x,y)$
$(0,1)$	1
$(1,0)$	$\frac{1}{2}$
$\left(\frac{1}{2}, \frac{1}{2}\right)$	1

. Thus, the minimum is $\frac{1}{2}$ and the maximum is 1.

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6 We use cylindrical coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $dx\,dy = r\,dr\,d\theta$

and $\displaystyle\iiint_V 4e^{x^2+y^2} z\,dV = \int_{z=0}^{2} \int_{r=0}^{\ln(2)} \int_{\theta=0}^{2\pi} 4z e^{r^2} r\,d\theta\,dr\,dz$

$$= \left\{\int_{\theta=0}^{2\pi} d\theta\right\} \left\{\int_{r=0}^{\sqrt{\ln(2)}} 2re^{r^2}\,dr\right\} \left\{\int_{z=0}^{2} 2z\,dz\right\}$$

$$= \{2\pi\} \left\{e^{r^2}\Big|_{r=0}^{\sqrt{\ln(2)}}\right\} \left\{z^2\Big|_{z=0}^{2}\right\} = 2\pi\{e^{\ln(2)} - 1\}\{4\}$$

$$= (2\pi)(1)(4) = \boxed{8\pi}.$$

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7 $\iint_{\sigma} \mathrm{curl}(\vec{F})\circ d\vec{S} = \oint_{\mathcal{C}} \vec{F}\circ d\vec{r}$, where $\mathcal{C}$ is the unit circle $x^2+y^2=1$, oriented counterclockwise. A parametrization of $\mathcal{C}$ is: $x=\cos(t)$, $y=\sin(t)$, $z=0$, $0 \xrightarrow{t} 2\pi$.

Thus, $\vec{F} = \langle \sin(t), \cos(t), 1\rangle$ on $\mathcal{C}$, and

$d\vec{r} = \langle -\sin(t), \cos(t), 0\rangle dt$, so

$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \int_{t=0}^{2\pi} \langle -\sin(t), \cos(t), 1\rangle \circ \langle -\sin(t), \cos(t), 0\rangle dt$$

$$= \int_{t=0}^{2\pi} 1\, dt = \boxed{2\pi}.$$

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8 $\iint\!\!\!\!\!\!\bigcirc_{\sigma} \vec{F}\circ d\vec{S} = \iiint_{\mathcal{N}} \mathrm{div}(\vec{F})\,dV = \int_{x=0}^{1}\int_{0}^{1}\int_{z=0}^{1} \cos^2(z)+\sin^2(z)+1\, dz\,dy\,dx$

$$= \int_0^1\int_0^1\int_0^1 2\,dz\,dy\,dx = \boxed{2}$$