

Fourth Examination
Tuesday, November 15, 2016

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 20 points.

1. Find the surface area of the portion of the surface $x = u$, $y = u$, $z = u + v^2$ corresponding to the rectangle $0 \leq u \leq 1$, $0 \leq v \leq 2$ in parameter space.
2. Compute $\int \int \int_{\mathcal{V}} x dV$, where \mathcal{V} is the region in the first orthant above the unit rectangle $0 \leq x \leq 1$, $0 \leq y \leq 1$ and below the plane given by $y + z = 1$.
3. Compute $\int \int \int_{\mathcal{V}} e^{-(x^2+y^2+z^2)^{3/2}} dV$, where \mathcal{V} is the upper half of the unit ball $x^2 + y^2 + z^2 \leq 1$.
4. Compute the work done when a particle moves along the path \mathcal{C} defined by $x = t$, $y = \frac{1}{2}t^2$, $z = \frac{1}{3}t^3$, $1 \leq t \leq 2$ through the force field defined by $\vec{F}(\vec{r}) = (1, \frac{1}{x}, \frac{1}{x^2})$.
5. Consider the vector field defined by $\vec{F}(x, y) = (y + 2x, x + 1)$.
 - (a) Is \vec{F} conservative? If so, compute a potential function for it.
 - (b) What is the work done by the vector field on a particle moving along the path $\mathcal{C} = (\cos(t), \sin(t))$, $t : 0 \rightarrow 2\pi$?
 - (c) What is the work done by the vector field on a particle moving along the path $x = t + t^{10} - t^5$, $y = t^{20} - t^{30}$, $t : 0 \rightarrow 1$?