Math. 302-05
Fall, 2016
R. B. Kearfott

**Third Examination**

*Thursday, October 20, 2016*

**Instructions:** This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 16 points, and 4 points are free

1. Find the directional derivative $D_u f(1,2,0)$ of $f(x,y) = x^2 + xy + y^2 - z$ in the direction of $\boldsymbol{u} = (1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$.

2. Find an equation for the tangent plane and two normal vectors to the graph of $f(x,y) = x\sin(xy)$ at the point $(x,y) = (1,\pi)$.

3. Locate all relative maxima, minima, and saddle points of

$$f(x,y,z) = \frac{1}{2}x^2 - xy - x + y.$$

   (In other words, find and classify all critical points of $f$.)

4. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = x^2 - y^2$ subject to $x^2 + y^2 = 9$.

5. Compute $\displaystyle\iint_{\mathcal{R}} xy dA$, where $\mathcal{R}$ is the region in the first quadrant between $y = x$ and $y = \sqrt{x}$.

6. Find $\displaystyle\iint_{\mathcal{R}} \sqrt{9 - x^2 - y^2} dA$, where $\mathcal{R}$ is the region in the first quadrant within the circle $x^2 + y^2 = 9$.