Math. 302-05
Fall, 2016
R. B. Kearfott

Final Examination

Friday, December 9, 2016, 11:00AM to 1:30PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 12 points, and 4 points are free.

1. Decompose the vector $\boldsymbol{u} = \langle -1, 1, 2\rangle$ into $\boldsymbol{u} = \boldsymbol{v} + \boldsymbol{w}$, where $\boldsymbol{v}$ is in the direction of $\boldsymbol{b} = \langle 1, 1, 1\rangle$ and $\boldsymbol{w}$ is orthogonal to $\boldsymbol{v}$.
2. Write down an equation for the plane that passes through the points $P(1, -1, 1)$, $Q(1, 0, 1)$ and $R(1, 2, 3)$.
3. Find parametric equations for the line tangent to the graph of $\boldsymbol{r}(t) = \langle \ln(t), t, e^{t-1}\rangle$ at $t = 1$.
4. Find the directional derivative $D_u f(1, 0, 2)$ of $f(x, y) = x + xy + y^2 - z^2$ in the direction of $\boldsymbol{u} = \langle 1/\sqrt{2}, 0, -1/\sqrt{2}\rangle$.
5. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to $x + y = 1$, $0 \le x \le 1$.
   Hint: In addition to using Lagrange multipliers to find constrained critical points, check the end points of the feasible set.
6. Compute $\displaystyle\int\int\int_{\mathcal{V}} 4ze^{x^2+y^2} dV$, where $\mathcal{V}$ is the cylinder with axis the z-axis, extending from $z = 0$ to $z = 2$, and with cross-section $x^2 + y^2 \le \ln(2)$.
7. Use Stokes' theorem to compute $\int\int_\sigma \mathrm{curl}(\vec{F}) \cdot d\vec{s}$, where σ is that half of the unit sphere $x^2 + y^2 + z^2 = 1$ corresponding to $z \ge 0$, oriented upward, and where $\vec{F}(x, y, z) = \langle -y, x, e^{\sin(z^7)}\rangle$.
8. Use the divergence theorem to compute the flux of the vector field $\vec{F}(x, y, z) = \langle x\cos^2(z), y\sin^2(z), z\rangle$ out of the unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.