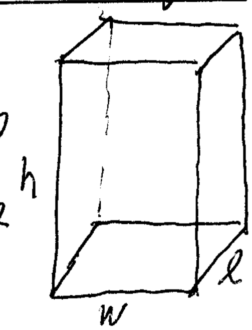
(1) $w\ell h = 2000$

$\min f = (.05)(2)wh + (.05)(2)\ell h + (.1)(2)w\ell$



We can either use substitution or Lagrange multipliers. We choose to substitute $h = 2000/w\ell$ to obtain:

$\min f(w,\ell) = \frac{200}{\ell} + \frac{200}{w} + 0.2w\ell$. We must have

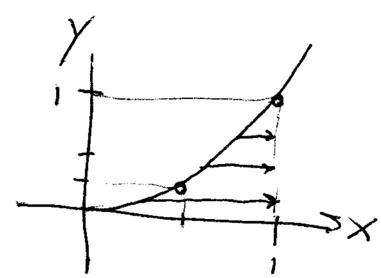
$\frac{\partial f}{\partial w} = \frac{-200}{w^2} + 0.2\ell = 0$; $\frac{\partial f}{\partial \ell} = -\frac{200}{\ell^2} + 0.2w = 0$

This gives $w = 1000/\ell^2$, $\ell = 1000/w^2$, whence

$w = w^4/1000 \Leftrightarrow w\left(1 - \frac{w^3}{1000}\right) = 0 \Rightarrow w = 0$ or $\boxed{w = 10}$

Hence, $w = 10$ and $\ell = 1000/10^2 = \boxed{10 = \ell}$. This gives $h = \frac{2000}{100} = \boxed{20 = h}$.

(2) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3}\,dx\,dy = \int_{x=0}^1 \int_{y=0}^{x^2} \sqrt{2+x^3}\,dy\,dx$



$= \int_{x=0}^1 x^2\sqrt{2+x^3}\,dx$ $\quad \boxed{u = 2+x^3;\ \frac{du}{3} = x^2dx}$

$= \frac{1}{3}\int_{u=2}^3 u^{1/2}du = \frac{2}{3}\left(\frac{1}{3}\right)u^{3/2}\Big|_{u=2}^3 = \boxed{\frac{2}{9}\left(3^{3/2} - 2^{3/2}\right)}$

(3) We use spherical coordinates. $dV = \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$

$0 \le \rho \le 3$, $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi/2$, so the integral reduces to

$\int_{\rho=0}^3 \int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/2} \rho(\rho^2\sin(\varphi))\,d\varphi\,d\theta\,d\rho$

$= \int_{\rho=0}^3 \int_{\theta=0}^{2\pi} \rho^3(1)\,d\theta\,d\rho = 2\pi\int_{\rho=0}^3 \rho^3 d\rho = \frac{2\pi}{4}3^4 = \boxed{\frac{81}{2}\pi}$

(4) $x(t) = 3\cos(t) + 1,\ y(t) = 3\sin(t) + 2;\ 0 \le t \le 2\pi.$
(There are many others.)

---

(5) (a) $(1+t) + (5+2t) + (-7+t) = 1$
$4t - 1 = 1 \Rightarrow 4t = 2 \Rightarrow \boxed{t = 1/2}$ at point $\left(\frac{3}{2}, 6, -\frac{13}{2}\right)$

(b) speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = \sqrt{1^2 + 2^2 + 1^2}$
$= \boxed{\sqrt{6}\text{ meters per second}}$

---

(6) We can use the divergence theorem:

$$\iint_{S} \vec{F} \cdot d\vec{A} = \iiint_{W} \operatorname{div}(\vec{F})\, dV = \int_{r=0}^{2} \int_{\theta=0}^{2\pi} \int_{z=0}^{3} (1+1+0)\, r\, dz\, d\theta\, dr$$
$$= 2(\pi 2^2)(3) = \boxed{24\pi}.$$

---

(7) We can use Stokes' Theorem:

$$\int_{C} \vec{F} \cdot d\vec{r} = \iint_{S} \operatorname{curl}(\vec{F}) \cdot d\vec{A},$$ where S is any surface bounded by C. However,

$$\operatorname{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\vec{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\vec{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\vec{k}$$
$$= (x-x)\vec{i} + (y-y)\vec{j} + (z-z)\vec{k}$$
$$= \vec{0}.$$

Hence $\int_{C} \vec{F} \cdot d\vec{r} = \boxed{0}$
