1 (a) We use the ratio test $\dfrac{a_{n+1}}{a_n} = \dfrac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \dfrac{2^{n+1}}{2^n}\dfrac{n!}{(n+1)!} = \dfrac{2}{n+1} \to 0 < 1$ as $n \to \infty$.

Therefore, $\sum_{n=0}^{\infty} \dfrac{2^n}{n!}$ converges by the ratio test.

(b) $\dfrac{1}{n^2 \ln x} \le \dfrac{1}{n^2}$ for $n \ge 2$, and $\sum_{n=2}^{\infty} \dfrac{1}{n^2}$ converges,

so $\sum_{n=2}^{\infty} \dfrac{1}{n^2 \ln(x)}$ converges by the comparison test.

(c) $|a_n| = \left|\dfrac{(-1)^n}{\ln(n)}\right| = \dfrac{1}{\ln(n)} \searrow 0$ as $n \to \infty$. (That is, the absolute values of the terms decrease to 0.) Therefore, the series converges by the alternating series test.

(d) We will use the ~~integral~~ comparison test. $\dfrac{1}{\ln(n)} > \dfrac{1}{n}$ and

$\sum_{n=2}^{\infty} \dfrac{1}{n}$ diverges by the p-series test. Therefore,

$\sum_{n=2}^{\infty} \dfrac{1}{\ln(n)}$ diverges by the comparison test.

2 (a) The center is $x_0=0$. The ratio of coefficients is
$$\left|\frac{a_n}{a_{n+1}}\right| = \frac{\frac{1}{n!}}{\frac{1}{(n+1)!}} = n+1 \to \infty \text{ as } n\to\infty.$$ Therefore, the radius of convergence is infinite, that is, the series converges for all x. Since the series converges everywhere, there are no end points to analyze.

(b) The ratio of absolute values of ~~terms~~ in coefficients is:
$$\frac{|a_n|}{|a_{n+1}|} = \frac{2^n}{2^{n+1}} = \frac{1}{2} = r.$$
The center is 3, so the series converges for $x\in(2.5, 3.5)$
The series becomes $\sum_{n=0}^{\infty} 2^n\left(-\frac{1}{2}\right)^n = \sum_{n=0}^{\infty}(-1)^n$, not convergent at $x=2.5$. Similarly, at $x=3.5$, the series becomes $\sum_{n=0}^{\infty} 1$, not convergent.

(c) The ratio of absolute values of coefficients is
$$\frac{|a_n|}{|a_{n+1}|} = \frac{\frac{1}{n}}{1/n+1} = \frac{n+1}{n} = 1+\frac{1}{n} \to 1 \text{ as } n\to\infty,$$ and the center of convergence is $x_0=1$. Therefore, the series converges for $|x-1|<1$, that is $0<x<2$. At $x=0$, the series becomes $\sum_{n=0}^{\infty}\frac{(-1)^{2n}}{n+1} = \sum_{n=0}^{\infty}\frac{1}{n+1}$, which diverges by the p-series test (with the substitution $m=n+1$). At $x=2$, the series becomes the alternating series $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$, which converges by the alternating series test. Therefore, $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}(x-1)^n$ converges for

$$\boxed{x\in(0,2]}$$

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2 d The ratio of terms is $\frac{a_n}{a_{n+1}} = \frac{n!}{(n+1)!} = \frac{1}{n+1} \to 0$ as $n \to \infty$.

Thus, the radius of convergence is 0, that is, the series converges for no x except at its center $x=0$.
(All terms are equal to 0 at $x=0$.)