Math. 301-03
Fall, 2011
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Final Examination

Monday, December 5, 2011, 11:00-13:30

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Point values are as indicated, and two points are free. You may keep this exam sheet.

1. Compute the following. (That is, write the quantities down in closed form.) Give exact answers, rather than decimal approximations. Show all of your work. (4 points apiece)

   (a) $\displaystyle \frac{d}{dx}\int_0^x \ln\left(\cos\left(e^{-\sin(t)}\right)\right)dt$ (b) $\displaystyle \int \frac{1}{x^2+3x+2}dx$ (c) $\displaystyle \int_0^{\sqrt[3]{\pi}} x^2\sin(x^3)dx$

   (d) $\displaystyle \int_{-\pi}^{\pi} x\sin(x)dx$ (e) $\displaystyle \int_{x=0}^{\sqrt{2}/2} \frac{dx}{\sqrt{1-x^2}}$ (f) $\displaystyle \int_0^{\infty} xe^{-x^2}dx$

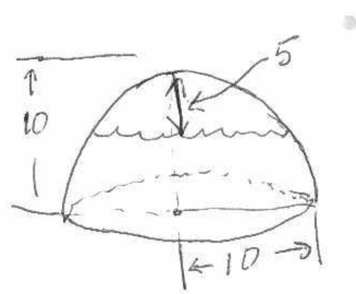
   (g) $\displaystyle \int_{-1}^{1} \frac{dx}{\sqrt{1-x}}$

2. A large freight train pulls out of the station and proceeds along a straight track with a constant acceleration a, for 15 minutes (that is for 0.25 hours). After the 15 minutes, it has gone 4 miles. (5 points apiece)
   (a) Compute a in miles per hour per hour.
   (b) How fast in miles per hour is the train going at the end of the 15 minutes?

3. Which of the following integrals converge and which diverge? In each case, explain why. (5 points apiece.)

   (a) $\displaystyle \int_1^{\infty} \frac{\sin^2(x)}{x^2}dx$ (b) $\displaystyle \int_0^1 \frac{1}{x^2}dx$

4. An oil storage tank is in the shape of an inverted hemisphere of radius 10 feet. How many cubic feet of oil are in the tank if it is filled to within 5 feet of the top? See the figure. (10 points)



5. An electrical cable is hung between two towers spaced 1 unit apart. The cable sags about 0.1276 units midway between the towers. It is well-known that such hanging cables follow a *catenary* curve, in the shape of the cosh function. If we say the cable stretches between $x = 0$ and $x = 1$, suppose the shape of this particular catenary curve is

$$y = \cosh(x - 0.5).$$

Find the total length of cable between the two towers. (10 points)

6. Write down the Taylor polynomial of degree 3 expanded about $a = 0$ for the function

$$f(x) = \int_{t=0}^{x} \frac{\sin(t)}{t}\,dt.$$

Evaluate this polynomial at $x = 0.1$. (15 points)

7. Consider the series

$$S = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n} (x - 1)^n.$$

(a) Compute the radius of convergence, and write down the open interval in which the series converges. (10 points)

(b) Check each end point of the interval of convergence to see whether or not the series converges there. (5 points)