

$$\textcircled{1} \textcircled{a} \sum_{k=1}^5 (2k+1) = 2 \sum_{k=1}^5 k + \sum_{k=1}^5 1 = 2 \frac{(5)(6)}{2} + 5 = \boxed{35}$$

$$\textcircled{b} \sum_{k=1}^5 f(x_k) \Delta x = \sum_{k=1}^5 (2k+1) = \boxed{35}$$

$$\textcircled{c} \sum_{k=1}^{30} \left(\frac{1}{k} - \frac{1}{k+1} \right) = 1 - \frac{1}{31} = \boxed{\frac{30}{31}}$$

$$\textcircled{2} \textcircled{a} \int u^2 + 1 du = \boxed{\frac{u^3}{3} + u + C}$$

$$\textcircled{b} \int \frac{1}{x^2+1} dx = \boxed{\arctan(x) + C}$$

$$\textcircled{c} \int (\sin(x))^2 + 1 \cos(x) dx \quad \boxed{u = \sin(x)}$$

$$= \int u^2 + 1 du = \frac{u^3}{3} + u + C \quad \boxed{du = \cos(x) dx}$$

$$= \boxed{\frac{(\sin(x))^3}{3} + \sin(x) + C}$$

$$\textcircled{3} \textcircled{a} \int_0^{\pi/2} \{ (\sin(x))^2 + 1 \} \cos(x) dx = \left[\frac{(\sin(x))^3}{3} + \sin(x) \right]_{x=0}^{\pi/2}$$

$$= \left[\frac{1}{3} + 1 \right] - \left[\frac{0^3}{3} + 0 \right] = \boxed{\frac{4}{3}}$$

$$\textcircled{b} \int 2te^{t^2} dt = \int e^u du = e^u + C = e^{t^2} + C, \quad \boxed{u = t^2}$$

$$\text{so } \int_0^{\sqrt{\ln(2)}} 2te^{t^2} dt = \int_0^{\ln(2)} 2te^{t^2} dt = \int_0^{\ln(2)} e^u du$$

$$= \left[e^u \right]_{u=0}^{\ln(2)} = e^{\ln(2)} - e^0 = 2 - 1 = \boxed{1}$$

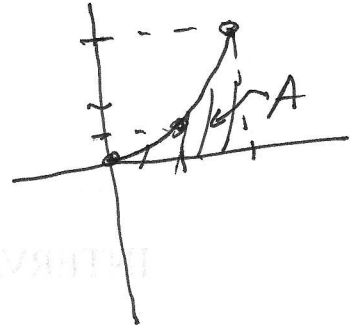
$$\textcircled{c} \int_{-1}^1 x^2 dx = \frac{x^3}{3} \Big|_{x=-1}^1 = \frac{1}{3} - \left(-\frac{1}{3} \right) = \boxed{\frac{2}{3}}$$

$$\textcircled{d} \int_0^1 x^2 dx = \frac{x^3}{3} \Big|_{x=0}^1 = \frac{1}{3} - \frac{0}{3} = \boxed{\frac{1}{3}}$$

④ ~~the~~ (see next page)

(4) The area is

$$A = \int_0^1 x^2 dx = \left[\frac{1}{3} \right]$$



INTERVAL ANALYSIS AND CONSTRUCTIVE
MATHEMATICS

Faculty of Mathematics, University of Cambridge, Cambridge, United Kingdom
Department of Mathematics, University of Cambridge, Cambridge, United Kingdom
Faculty of Mathematics, University of Cambridge, Cambridge, United Kingdom
Department of Mathematics, University of Cambridge, Cambridge, United Kingdom

11-11-17, 11:11

1. Properties of the field

1.1. Construction of the field

In this section we prove that the real numbers form a field. We start by showing that the real numbers are a commutative ring. This is done by showing that the real numbers are a complete metric space and that the addition and multiplication of real numbers are continuous. The next step is to show that the real numbers have no zero divisors. This is done by showing that the real numbers are an integral domain. Finally, we show that the real numbers have a multiplicative inverse for every non-zero element. This is done by showing that the real numbers are a field.