Fourth exam answers, page ①, Spring, 2017 Math 270-10

① ⓐ $\sum_{k=1}^{5}(2k+1) = 2\sum_{k=1}^{5}k + \sum_{k=1}^{5}1 = 2\frac{(5)(6)}{2} + 5 = \boxed{35}$

ⓑ $\sum_{k=1}^{5} f(x_k)\Delta x = \sum_{k=1}^{5}(2k+1) = \boxed{35}$

ⓒ $\sum_{k=1}^{30}\left(\frac{1}{k} - \frac{1}{k+1}\right) = 1 - \frac{1}{31} = \boxed{\frac{30}{31}}$

② ⓐ $\int u^2+1\,du = \boxed{\frac{u^3}{3}+u+C}$

ⓑ $\int \frac{1}{x^2+1}dx = \boxed{\arctan(x)+C}$

ⓒ $\int([\sin(x)]^2+1)\cos(x)\,dx$

$\boxed{\begin{array}{l} u = \sin(x) \\ du = \cos(x)dx \end{array}}$

$= \int u^2+1\,du = \frac{u^3}{3}+u+C$

$= \boxed{\frac{[\sin(x)]^3}{3} + \sin(x) + C}$

③ ⓐ $\int_0^{\pi/2}\{[\sin(x)]^2+1\}\cos(x)\,dx = \left[\frac{(\sin(x))^3}{3} + \sin(x)\right]_{x=0}^{\pi/2}$

$= \left[\frac{1}{3}+1\right] - \left[\frac{0^3}{3}+0\right] = \boxed{\frac{4}{3}}$

ⓑ $\int 2te^{t^2}dt = \int e^u du = e^u + C = e^{t^2}+C,$

$\boxed{\begin{array}{l} u = t^2 \\ du = 2t\,dt \end{array}}$

so $\int_0^{\sqrt{\ln(2)}} 2te^{t^2}dt = \int_0^{\sqrt{\ln 2}} 2te^{t^2}dt$.

$= e^{t^2}\Big]_{t=0}^{\sqrt{\ln(2)}} = e^{\ln(2)} - e^0 = 2-1 = \boxed{1}$

ⓒ $\int_{-1}^{1} x^2dx = \frac{x^3}{3}\Big|_{x=-1}^{1} = \frac{1}{3} - \left(\frac{-1}{3}\right) = \boxed{\frac{2}{3}}$

ⓓ $\int_0^1 x^2dx = \frac{x^3}{3}\Big|_{x=0}^{1} = \frac{1}{3} - \frac{0}{3} = \boxed{\frac{1}{3}}$

④ ~~The~~ (see next page)

(4) The area is

$$A = \int_0^1 x^2 \, dx = \boxed{\frac{1}{3}}.$$