First Examination
Monday, February 6, 2017

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 20 points.

1. Compute the following limits, if they exist, and state when they do not exist.

   (a) $\displaystyle\lim_{x\to1}\frac{x^2-1}{x^2+1}$
   (b) $\displaystyle\lim_{x\to1}\frac{x-1}{x^2-1}$
   (c) $\displaystyle\lim_{x\to+\infty}\frac{x^3+x+1}{3x^3-x-1}$

   (d) $\displaystyle\lim_{x\to-\infty}\frac{x^3+x+1}{3x^3-x-1}$
   (e) $\displaystyle\lim_{x\to+\infty}\frac{x^3+x+1}{3x^2-x-1}$
   (f) $\displaystyle\lim_{x\to-\infty}\frac{x^3+x+1}{3x^2-x-1}$

2. Suppose $\lim_{x\to1} f(x) = 4$, $\lim_{x\to4} g(x) = +\infty$, and $\lim_{x\to+\infty} h(x) = 10$. Compute the following limits.

   (a) $\displaystyle\lim_{x\to1}\sqrt{f(x)}$
   (b) $\displaystyle\lim_{x\to1}\frac{1}{g(f(x))}$
   (c) $\displaystyle\lim_{x\to4}h((g(x)))$

3. Compute the following limits, if they exist, and state if they do not.

   (a) $\displaystyle\lim_{x\to0}\frac{\sin(2x)}{\sin(3x)}$
   (b) $\displaystyle\lim_{x\to\pi/2}\tan(x)$
   (c) $\displaystyle\lim_{x\to\pi/2}\tan^2(x)$

   (d) $\displaystyle\lim_{x\to+\infty}\tan^{-1}(x)$
   (e) $\displaystyle\lim_{x\to-\infty}\tan^{-1}(x)$

4. Compute $\cos(\sin^{-1}(3/5))$. Your answer must be expressed as an exact fraction.

5. Find the exact values of the following expressions, without using a calculator.

   (a) $\ln(e^2)$
   (b) $e^{\ln(3^2+3+1)}$
   (c) $\ln(\sqrt{e^{3/5}})$