Math. 270-03
Spring, 1997
R. B. Kearfott

## Third Examination

Wednesday, April 2, 1997
Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 20 points.

1. Suppose that the mileage that an automobile gets decreases gradually as the automobile engine ages, according to the following table:

| Mileage | Miles per gallon | Mileage | Miles per gallon |
| ---: | :---: | ---: | :---: |
| 0 | 35 | 60,000 | 30 |
| 20,000 | 33 | 80,000 | 27 |
| 40,000 | 32 | 100,000 | 24 |

Based on this table, give lower and upper bounds on the total amount of gasoline the automobile would use if it were driven 100,000 miles. Hint: You should first convert miles per gallon to gallons per mile.
2. If $f$ is as in Figure 1, then give an approximation to

$$
\int_{0}^{8} f(x) d x
$$

3. Find the average value of $f(t)=1+2 t$ over the interval $t=0$ to $t=2$.
4. If $F(x)=x^{4} / 4$, then it is known that $F^{\prime}(x)=x^{3}$. Use this fact to compute the following integrals.

$$
\text { (a) } \quad \int_{-1}^{1} x^{3} d x \quad \text { (b) } \quad \int_{0}^{2} x^{3} d x
$$

5. Compute left and right Riemann sums to

$$
\int_{-1}^{1} x^{3} d x
$$

with 4 subintervals of length $\frac{1}{2}$. Compare the result to your result in Problem 4.


Figure 1: The function $f$ for Problem 2

