1 (a) $f'(x) = \cos(x^2)(2x)$ using the chain rule.

(b) $f'(x) = \dfrac{2x(x^3+1) - (x^2+1)(3x^2)}{(x^3+1)^2}$ using the quotient rule.

$= \dfrac{-x^4 - 3x^2 + 2x}{(x^3+1)^2}$ (simplifying). $x \neq -1$.

(c) $f'(x) = \dfrac{1}{x^2 \sin(x)}(2x \sin x + x^2 \cos x)$

using the product rule and the chain rule.

2 We have $T(t) = T(h(t))$, so

$\dfrac{dT}{dt} = \dfrac{dT}{dh}\dfrac{dh}{dt}$ $\dfrac{dT}{dh} = \dfrac{-.7}{2\sqrt{h}} = -.35h^{-1/2}$.

$\dfrac{dh}{dt} = +\dfrac{10000}{60} e^{-t/60}$ (by the chain rule),

so $\dfrac{dT}{dt} = \left[(-.35)h^{-1/2}\right]\left[\dfrac{10000}{60} e^{-t/60}\right]$.

(a) $h(60) = 10000(1 - e^{-60/60}) = 10000(1 - 1/e) \approx 6321$ ft.

(b) $\left.\dfrac{dT}{dt}\right|_{t=60} \approx \left[\left.\dfrac{dT}{dh}\right|_{h=6321}\right] \cdot \left[\left.\dfrac{dh}{dt}\right|_{t=60}\right]$

$\approx \left(\dfrac{-.35}{\sqrt{6321}}\right)\left(\dfrac{10000}{60} e^{-1}\right) \approx -.27$ degrees per minute.

3 $f^{-1}(f(x)) = x$, so $\left.\frac{df^{-1}}{dy}\right|_{y=f(x)} \frac{df}{dx} = 1$, where $x = f^{-1}(y)$,

so $f^{-1\prime}(y) = \dfrac{1}{f'(f^{-1}(y))}$.

We first obtain an expression for f^{-1}:

$y = (\sin(x))^3 \Leftrightarrow y^{1/3} = \sin x$, so $x = \arcsin(y^{1/3})$.

Thus, when $y = (\sqrt{2}/2)^3$, $x = \arcsin(\sqrt{2}/2) = \pi/4$.

We have $f'(x) = 3(\sin(x))^2(\cos(x))$, so

$$\cancel{f^{-1\prime}(\pi/4)} \quad f^{-1\prime}\left(\left(\frac{\sqrt{2}}{2}\right)^3\right) = \frac{1}{3\left[\sin(\pi/4)^2\cos(\pi/4)\right]}$$

$$= \frac{1}{3\,(\sqrt{2}/2)^2\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{1}{3\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)} = \boxed{\frac{2\sqrt{2}}{3}}.$$

4 (a) $f'(x) = x^2 - 1$. For $x \in [-1,1]$, $f'(x) \le 0$, so f is decreasing on $[-1,1]$.

(b) $g'(x) = \frac{4}{3}x^3 - 1 \ge x^2 - 1$ for $x \ge 1$. Therefore, since $g'(x) \ge f'(x)$ for $x \ge 1$, $g(x) \ge f(x)$ for $x \ge 1$.