## Second Examination

Wednesday, February 22, 2006

Instructions: This exam should be done on your own paper. The answers should be written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 25 points.

1. Examining figure 1, answer the following questions.


Figure 1: The figure for Problem 1
(a) At which points is $f$ increasing?
(b) At which points is $f$ decreasing?
(c) At which points is $f^{\prime}$ increasing?
(d) At which points is $f^{\prime}$ decreasing?
(e) At which points is $f^{\prime}$ positive?
(f) At which points is $f^{\prime}$ negative?
(g) At which points is $f^{\prime}$ approximately equal to 0 ?
(h) From the points where $f^{\prime}$ is approximately zero, at which of them is $f^{\prime \prime}$ positive?
(i) From the points where $f^{\prime}$ is approximately zero, at which of them is $f^{\prime \prime}$ negative?
2. Draw pictures of plausible graphs of both $s(t)$ and $s^{\prime}(t)$, for the following situation: A skater races for two laps around an oval rink, speeding up in the straight sections and slowing down on the curved ends.
3. Suppose $f^{\prime \prime}(x)=2 x-1$.
(a) For what values of $x$ is $f$ concave up?
(b) For what values of $x$ if $f$ concave down?
(c) Draw a possible graph of $f^{\prime}(x)$.
4. When you breathe, a muscle (called the diaphragm) reduces the pressure around your lungs and they expand to fill with air. The table shows the volume of a lung as a function of the reduction in pressure from the diaphragm. Pulmonologists define the compliance of the lung as the derivative of this function.

| Pressure reduction in <br> cm of water | Volume in liters |
| :---: | :---: |
| 0 | 0.20 |
| 5 | 0.29 |
| 10 | 0.49 |
| 15 | 0.70 |
| 20 | 0.86 |
| 25 | 0.95 |
| 30 | 1.00 |

(a) What are the units of compliance?
(b) Estimate the maximum compliance of the lung.
(c) Explain why the compliance gets small when the lung is nearly full (around one liter).

