SEPARATION OF VARIABLES VIA SINC CONVOLUTION

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Abstract

In Vol 1. of their 1953 text, Morse and Feshbach prove for the case of 3-dimensional Poisson and Helmholtz PDE that separation of variables is possible for essentially 13 different types of coordinate systems. A few of these (rectangular, cylindrical, spherical) are taught in our undergraduate engineering-math courses. We sketch a proof that one can ALWAYS achieve separation of variables when construction a particular solution of any one of the following problems, via use of Sinc convolution:

$$\Delta\, u = -f(\bar{r}) \quad \bar{r} \in B\,; \tag{1}$$

or

$$\frac{\partial u}{\partial t} - \Delta\, u = f(\bar{r}, t) \in B \times (0, T)\,; \tag{2}$$

or

$$\frac{\partial^2 u}{\partial t^2} - \Delta\, u = f(\bar{r}, t) \in B \times (0, T)\,, \tag{3}$$

with B any curvilinear region in $\mathbb{R}^n$, with Δ the Laplacian in $\mathbb{R}^n$, n = 1 , 2 , 3 , under the assumption that calculus is used to model f and the boundary of the region B . That is, one can solve each of these problems via use of one dimensional Sinc matrices of order m. We can thus circumvent the use of large matrices to get an approximate solution that is uniformly accurate to within an error of the order of $\exp\left(-c\, m^{1/2}\right)$, where c is a positive constant independent of m or n. Indeed, the function f can also depend on u and $\nabla\, u$ for many cases of (1), on u and $\nabla\, u$ for all cases of (2) , and on u, $\nabla\, u$ and u_t for all cases of (3) .