Erratum to Rigorous Global Search, Chapter 1

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In the following, “Line +” means the numbered line from the top, while “Line -” means the numbered line from the bottom. The text before the symbol “→” is that I think to be possibly mistakenly printed, while the text after the symbol “→” is that I think to be possibly correct.

1. Page 4, Line +8 : + → −
2. Page 7, Line +19: \([0.1346, 0.4664] \subset [0.134, 0.475] \rightarrow [0.1346, 0.4774] \subset [0.134, 0.478]
3. Page 12, Line +4 : \(x \subset \mathbb{IR} \rightarrow x \in \mathbb{IR}
4. Page 14, Line -5 : \(f_u(X) \rightarrow F_u(X)
   \text{Line -7 : } f \rightarrow F
5. Page 16: The contents of the Line +1 through Line +17 were rewritten as the following.

For example, suppose \(\phi(x) = x^2 - x\), \(x = [-3/8, 3/4]\) and \(\bar{x} = 1/2\), so that \(\nabla \phi(x) = 2x - 1 = [-7/4, 1/2]\) is the exact range of the derivative of \(\phi\) over \(x\). Then the natural interval extension obtained from the particular expression \(x^2 - x\) is \(\phi_n([-3/8, 3/4]) = [-3/4, 15/16]\), while the mean value extension is

\(\phi_2([-3/8, 3/4], 1/2) = \frac{1}{4} + [-7/4, 1/2]([-3/8, 3/4] - \frac{1}{2}) = [-11/16, 41/32]\).

Thus \(w(\phi_n([-3/8, 3/4])) = 1.68750\) and \(w(\phi_2([-3/8, 3/4], 1/2)) = 1.96875\), whereas the range is \(\phi^u([-3/8, 3/4]) = [-1/4, 33/64]\) and its width is \(w(\phi^u([-3/8, 3/4])) = 0.76563\). This illustrates that a second order extension may not be superior to a first order extension (and may actually be substantially wider than a natural extension) when the widths of the arguments are large. It also illustrates that the extensions are different: \(\phi_n([0, 1]) \cap \phi_2([0, 1], 1/2) = [-3/4, 15/16] \cap [-11/16, 41/32] = [-11/16, 15/16]\) is also an enclosure for the range of \(\phi\) over \([-3/8, 3/4]\) that is sharper than either \(\phi_n([-3/8, 3/4])\) or \(\phi_2([-3/8, 3/4], 1/2)\) separately.
If on the other hand $x = [0.49, 0.51]$ and $\bar{y} = 0.5$, then the natural interval extension is $\phi_n([0.49, 0.51]) = [-0.2699, -0.2299]$, $\phi_2([0.49, 0.51], 0.5) = [-0.2502, -0.2498]$, $\phi''([0.49, 0.51]) = [-0.25, -0.2499]$, and a hint of the convergence behavior of $\phi_n$ and $\phi_2$ is seen.

6. Page 17, Line +4: $Y \subseteq X \rightarrow X \subseteq Y$
   Line -1: $x_0 = 1 \rightarrow x_0 = 0.1$

7. Page 22, Line +9: $c_{n,n} \rightarrow c_n$

8. Page 28, Line -13: $f_{s}(x) = f(0) + [1, 2](x - 1) \rightarrow f_{s}(x) = f(1) + [1, 3](x - 1)$
   Line -12: $f_{s}(x) = f(0) + [1, 2](x - 1) \rightarrow f_{s}(x) = f(1) + [1, 2](x - 1)$

9. Page 29: The contents of the Line +1 through the Line +5 were rewritten as follows.
   Theorem 1.10 is well known; its proof is not deep, and can be constructed from the following: $\mu \in S(f, x, \bar{y})$ implies

   $$\mu - f'(\bar{y}) = \frac{f(x) - f'(\bar{x})}{x - \bar{x}} = \frac{1}{2} f''(c_2)(x - \bar{x})$$

   for some $x \in X$ and $c_2 \in X$, while, for the same $x$, $f'(x) - f'(\bar{x}) = f''(c_1)(x - \bar{x})$ for some $c_1 \in X$. Thus,

   $$(\mu - f'(\bar{y}))/(f'(x) - f'(\bar{x})) = \frac{1}{2} f''(c_2) \approx \frac{1}{2},$$

   so $S(f, x, \bar{y}) \approx \frac{1}{2} f''(c_1)$, and $w(c[x, \bar{x}]) = c w([x, \bar{x}])$.

10. Page 29, Line -14: $[0, .04167] \rightarrow [0, .08334]$
    Line -13: $[1, 1.04167] \rightarrow [1, 1.08334]$
    Line -12: $A = ([1, 1.25], [0, .25]) \rightarrow A = ([1, 1.0834], [0, .0834])$

11. Page 30, Line -3: $x \subseteq \mathbb{IR}^2 \rightarrow x \in \mathbb{IR}^2$

12. Page 32, Line -11: $[-0.75, 0.75] \rightarrow [-0.625, 0.625]$

13. Page 32: The contents of the Line -8 through the Line -7 were rewritten as following.
   Although $\phi_{(1,2)}(X) = \phi_{(2,1)}(X)$ in some cases (see Exercise 1 on page 35), this is in general not true, as seen in Example 1.2.

15. Page 56, Line +7: \[ S(F, X, \bar{X}) \to S(F, X, \bar{X}) \]

16. * Page 57,

Line -5: \[
\begin{pmatrix}
-0.405, 0.405 \\
-0.005, 0.005
\end{pmatrix}
\to
\begin{pmatrix}
-0.505, 0.505 \\
-0.005, 0.005
\end{pmatrix}
\]

Line -4: \[
\begin{pmatrix}
-0.455, 0.355 \\
-0.005, 0.005
\end{pmatrix}
\to
\begin{pmatrix}
-0.555, 0.455 \\
-0.005, 0.005
\end{pmatrix}
\]

* Dr. R. Baker Kearfott, author of the book, thanks Sergey Shary for pointing out these two errors.