Errata for
Classical and Modern Numerical Analysis:
Theory, Methods, and Practice
(for the first printing)

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Chapter 1

p. 2, in the proof of Theorem 1.2: There is an implicit assumption that
\[ \int_a^b w(x)dx \neq 0, \] and the case \( \int_a^b w(x)dx = 0 \) is not considered. To have
the proof be complete, we need to observe that, if \( \int_a^b w(x)dx = 0 \), \( w(x) \)
must be identically equal to zero except on a set of measure zero. Thus,
\[ \int_a^b f(x)w(x)dx = 0, \] and Theorem 1.2 is also true in this special case.

p. 15, second line of Example 1.12: It should be \( fl \) instead of \( f \ell \).

p. 16, Theorem 1.8, its proof, and the definition of condition number:
There is confusion between \( x^* \) and \( x \), and the formula for the condition
number should only depend on one of these. The following modifications
will reduce the confusion:

\[ \left| \frac{f(x) - f(x^*)}{f(x)} \right| \approx \left| \frac{x f'(x)}{f(x)} \right| \frac{|x - x^*|}{x} \]

PROOF The linear Taylor approximation of \( f(x^*) \) about \( f(x) \) for small
values of \( |x - x^*| \) is given by \( f(x^*) \approx f(x) + f'(x)(x^* - x) \). Rearranging
the terms immediately yields the result.

We now define the condition number of a function \( f(x) \) as

\[ \kappa_f(x) := \left| \frac{x f'(x)}{f(x)} \right| \]
p. 19, Table 1.1: Although only 23 bits are used for single precision and 52 bits are used for double precision, $t = 24$ for single precision and $t = 53$ for double precision, because the first digit is assumed to be 1.

p. 20, Table 1.2: $\epsilon_m$ for double precision should be $2^{-53} + 2^{-105} \approx 1.11 \times 10^{-16}$, and for single precision should be $2^{-24} + 2^{-45} \approx 5.96 \times 10^{-8}$.

p. 30, problem 2: What was defined was not exactly the traditional sinc function. The first part of Problem 2 should instead read:

Write down a polynomial $p(x)$ such that $|S(x) - p(x)| \leq 10^{-10}$ for $-0.2 \leq x \leq 0.2$, where

$$S(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

Note: $\text{sinc}(x) = S(\pi x) = \sin(\pi x)/(\pi x)$ is the “sinc” function (well-known in signal processing, etc.).

p. 66, (2.27): Instead of

$$\frac{\hat{x}_k - x}{x_k - \alpha} \to 0 \text{ as } k \to \infty,$$

it should be

$$\frac{\hat{x}_k - \alpha}{x_k - \alpha} \to 0 \text{ as } k \to \infty.$$

Chapter 3

p. 101, lines 1 to 4 of the proof: The sentence should read: “The proof rests on the result in linear algebra that any square matrix is similar to an upper triangular matrix, i.e., given any $n$ by $n$ matrix $A$, there exists a unitary matrix $P$ such that $PAP^{-1} = \Lambda + U$, where $\Lambda$ is diagonal and $U$ is upper triangular with zeros on the diagonal.

p. 141, in (3.32): There is an extra “.”.

p. 143, line above (3.39): There should be no closing parentheses.

p. 154, line -8: It should be “$i = 1, 2, \ldots, n$”, rather than “1 = 1, 2, \ldots, n”.

p. 182, problem 23: Since an “operation” could mean a fused multiply-add, the phrasing of this question may cause some confusion. Also, it is unclear how to achieve $n^3$ multiplications. (See the analysis in the instructor’s answer guide.) Here is a suggested replacement for the problem:

Compute the number of multiplications it takes to compute the inverse of a matrix according to the note on page 110 as $cn^3 + O(n^2)$. (That is, determine $c$.)

*Hint: $c$ will be smaller if you take advantage of the fact that the right-hand-sides of the systems you are trying to solve are the unit vectors $e_j$.\)
p. 188, problem 55(b): For consistency of notation with the rest of the text, it should be “find $k$” and “$X_k$” rather than “find $t$” and “$x_t$.”

p. 188, problem 56(b): Add the following sentence before the parenthetical note: “Here, $y_0 = b$ and the $x_i$ are computed according to the Full Orthogonalization Method described on page 173.”

p. 201, line 2: There should not be a parenthesis after $(1, 0, 0)^T$.

Chapter 4

p. 220, in (4.19): It should be $f^{(2n)}(\xi)$ instead of $f^{2n}(\xi)$.

p. 236, last row of Table 4.2: This row is incorrect: $K$ and $M$ so $f = p + KM$ are not readily apparent for least squares, although bounds can be computed from the theory in this section.

p. 237, line 13: It should be “\sin([0, 0.05])” instead of “\sin([0,0,05])”.

p. 237, lines 7, 10, 11, 14, 15, etc. and p. 238, line 2, etc.: The last term in (4.26) should be $-\frac{1}{5040}x^7 \cos(\xi)$, rather than $-\frac{1}{5040}x^7 \sin(\xi)$. As a consequence, the numbers in the other indicated lines are incorrect. The page should be as follows:

$$\sin(x) \in x - \frac{x^3}{6} + \frac{x^5}{5!} - \frac{1}{5040}x^7 \cos(\xi) \quad \text{for some } \xi \in [-0.1, 0.1]. \quad (4.26)$$

We can replace $\cos(\xi)$ by an appropriate interval, say, by $[1 - x^2/2, 1]$, to get a pointwise estimate; for example,

$$\sin(0.05) \in .05 - \frac{.05^3}{6} + \frac{.05^5}{120} - \frac{.05^7}{5040} [0.99875, 1] \subseteq [0.04997916927067, 0.04997916927068],$$

where the above bounds are mathematically rigorous. Here, $K$ was evaluated at the point $x$, but, $\cos(\xi)$ was replaced by the aforementioned bounds. Similarly,

$$\sin(-0.01) \subseteq (-0.01) - \frac{(-.01)^3}{6} + \frac{(-.01)^5}{120} - \frac{(-.01)^7}{5040} [0.99995, 1] \subseteq [-0.009998333417, -0.0099998333416].$$

Thus, since we know $\sin(x)$ is monotonic for $x \in [-0.01, 0.05],$

$$[-0.0099998333417, 0.04997916927068]$$

represents a fairly sharp bound on the range $\{\sin(x) \mid x \in [-0.01, 0.05]\}$. Alternately, it may be more convenient in some contexts to evaluate $K$
and \( M \) over the entire interval, although this leads to a less sharp result. Using that technique, we would have

\[
\sin(0.05) \in 0.05 - \frac{0.05^3}{6} + \frac{0.05^5}{120} - \frac{0.05^7}{7!} [0.995, 1]
\]

\[
\subseteq 0.05 - \frac{0.05^3}{6} + \frac{0.05^5}{120} +
\]

\[
[-0.1550992063493, -0.154234871031748 \times 10^{-12}]
\]

\[
\subseteq [0.04997916927065, 0.04997916927071],
\]

and

\[
\sin(-0.01) \in (-0.01) - \frac{(-0.01)^3}{6} + \frac{(-0.01)^5}{5!} - \frac{(-0.01)^7}{7!} +
\]

\[
\frac{[-0.1, 0.1]^8}{8!} [-0.1, 0.1]
\]

\[
\subseteq (-0.01) - \frac{(-0.01)^3}{6} + \frac{(-0.01)^5}{5!} - \frac{(-0.01)^7}{7!} [0.995, 1]
\]

\[
\subseteq [-0.00999998333417, -0.00999998333416],
\]

thus obtaining (slightly less sharp) bounds

\[
[-0.00999998333417, 0.04997916927071].
\]

In general, substituting intervals into the polynomial approximation itself does not give sharp bounds on the range. For example,

\[
\sin([-0.01, 0.05]) \in ([-0.01, 0.05]) - \frac{([-0.01, 0.05])^3}{6} + \frac{([-0.01, 0.05])^5}{120}
\]

\[
- \frac{([-0.01, 0.05])^7}{7!} [0.995, 1]
\]

\[
\subseteq [-0.01002083333433, 0.0500016927084].
\]

**pp. 238–249:** Instances of “\( N \)” here occur in contexts where, previously, “\( n \)” occurred (for example, in Definition 4.12 on page 238, and in previous sections). In these pages, such instances of “\( N \)” should be changed to “\( n \)”.

**p. 243:** There should be a comma after the \((x - x_j - 2)^3\) in the second to the last line of (4.29).

**p. 256:** In Remark 4.43, it should be “Euler’s formula” instead of “Euler’s identity”.

**p. 284, problem 1(d):** it should be “\( \varphi_3 \equiv t^3 \)” , rather than “\( \varphi_3 \equiv x^3 \)”.

**p. 284, problem 4:** It should be “Use the Gram–Schmidt” instead of ”“Use Gram-Schmidt”.

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p. 286, problem 13: Since the \( L_k(x) \) in this problem are the same as the \( \ell_k(x) \) in (4.8) on page 212, \( L_k \) should be changed to \( \ell_k \), for consistency. Similarly, in problem 15 on the same page, \( l_i \) should be \( \ell_i \) for consistency.

p. 287, problem 18: The polynomial should be of degree 3, so it should be referenced as \( P_3(x) \), not as \( P_2(x) \).

p. 289, problem 32: As printed, \( f \) is not uniquely defined at \( x = \pi \). Replace “0 \( \leq x \leq \pi \)” by “0 \( \leq x < \pi \”).

p. 290, problem 38: Part (a) is assumed. Thus, part (a) should be absent, and the problem should be to prove that \( \varphi \) is constant on \([0, 1)\). It is also helpful to assume some kind of continuity, such as \( \lim_{x\to 0^+} \varphi(x) = \varphi(0) \). (Otherwise, \( \varphi \) could be set to an arbitrary value at a countable number of points, and still satisfy the recursion relation and orthogonality conditions with respect to Lesbegue measure.)

Chapter 5

p. 321, problem 16: It should be

\[
x_{k+1} = -(A - 3I)^{-1}(A - 5I)^{-1}x_k,
\]

instead of

\[
x_{k+1} = -(A + 3I)^{-1}(A - 5I)^{-1}x_k,
\]

p. 326, formula on line 2: Instead of:

\[
f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} + \frac{h}{2}f''(\xi(x)).
\]

it should be:

\[
f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2}f''(\xi(x)).
\]

p. 331, line 17: It should be “\( \cos(v_q)' - v_p' \)” rather than “\( \cos(v_q)' - v_p \)”.

p. 332, line -8: It should be “\( \partial f/\partial x_i \)” instead of “\( df/dx_i \)”.

p. 332, lines -3 and -2: It should be “\( \partial f/\partial x_1 \)” and “\( \partial f/\partial x_2 \)” instead of “\( df/dx_1 \)” and “\( df/dx_2 \)”.

Chapter 6

p. 346, first line of Corollary 6.1: It should be “\( \{p_i\}_{i\geq 0} \)” instead of “\( \{p\}_{i\geq 0} \)”.

p. 349, line 2: It should be “\( z_j, 0 \leq j \leq m \)” instead of “\( z_j, 0 \leq j \leq n \)”.  

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\textbf{p. 349, Table 6.1:} The indexing on the $\alpha$'s and $z$'s should go from 0 to $m$, rather than from 1 to $m + 1$, to be consistent with the rest of the text.

\textbf{p. 349, line 2 of Lemma 6.1:} It should be “of at most degree $2m+1$” instead of “of a most degree $2m + 1$”.

\textbf{p. 350, line -7:} It should be $f^{(2m+2)}(\xi)$ instead of $f^{2m+2}(\xi)$.

\textbf{p. 352, line -5:} It should be $\prod_{i=0 \atop i \neq j}^{m} (x_j - x_i)^2$ instead of $\prod_{i=0 \atop i \neq j}^{n} (x_j - x_i)^2$.

\textbf{p. 367, last line:} It should be “integrals over infinite intervals” instead of “in- tegrals over infinite integrals”.

\textbf{p. 377, problem 7:} The authors do not know a general closed-form formula for the $n$-th component in (6.14). A suggested rewriting of problem 7 is:

Compute the fourth and fifth components in (6.14), assuming order 4 Taylor arithmetic is to be used. Can a general formula be derived? Can a recursive routine be devised to compute the $n$-th term, without explicitly computing it by hand first?

Instructors can consult the solutions manual for an answer to this modified question.

\textbf{p. 377, problem 8:} There is a similar difficulty with this problem as with problem 7. It is suggested that the problem be replaced by:

Compute the components of the degree-4 Taylor object for $(u\nabla)^n$.

\textit{Project: Write a program that recursively computes a general coefficient for a degree $N$ Taylor object for $(u\nabla)^n$}.

\textbf{p. 378, problem 15:} This problem is erroneously stated. As a counterexample to part (a), take $f(x) = x^{10}$, and $a = 10$. Then, $f(a) + f(-a) = 2 \cdot 10^{10}$, while $\int_{-1}^{1} f(x) dx = 2/11$, and the error bound is only $(10^3 - 10^2 + \frac{1}{2}) \cdot 90$.

\textbf{p. 379, line 1:} For consistency with the rest of the text, “Trapezoid rule” should be “trapezoidal rule”.

\section*{Chapter 7}

\textbf{p. 383, line 10:} Instead of “on page 482 in Section 8.1,” it should be “on page 482 in Section 8.7.”

\textbf{p. 419, paragraph beginning on line 6:} The second sentence is erroneous. It should read

A spring is “stiff” if its damping constant is large; in such a mechanical system, motions of the spring will damp out fast relative to the time scale on which we are studying the system.
instead of

A spring is “stiff” if its spring constant is large; in such a mechanical system, the spring will cause motions of the system that are fast relative to the time scale on which we are studying the system.

p. 433, line 4 of problem 16: It should be “determine β” instead of “determine β”.

p. 435, problem 24: (i) Method (i) Should read

\[ y_{j+2} - 2y_{j+1} + y_j = 2h(f(t_{j+1}, y_{j+1}) - f(t_j, y_j)). \]

(iii) Method (iii) should read

\[ y_{j+1} - y_j = hf(t_{j+1}, y_{j+1}). \]

Finally, the precise problem being solved is irrelevant to consistency and stability of the method, so the first sentence of the problem should be deleted.

Chapter 8

p. 443, line -13: It should be “G(x(k))” rather than “G(x^k)”.

p. 445, last line: It should be “Φ’(s)” rather than “Φ’(s)”.

p. 446, lines 6 and 8: the derivative signs in the Leibnitz notation should be partial derivative signs. That is, lines 6, 7, and 8 should read:

\[
G'(x) = \begin{pmatrix}
\frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\
\frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{3}x_2 \sin(x_1x_2) & \frac{1}{3}x_1 \sin(x_1x_2) \\
x_2 e^{-x_1x_2} & \frac{x_1}{20} e^{-x_1x_2}
\end{pmatrix}
\]

and

\[
\left| \frac{\partial g_1}{\partial x_1} \right| \leq \frac{1}{3}, \quad \left| \frac{\partial g_1}{\partial x_2} \right| \leq \frac{1}{3}, \quad \left| \frac{\partial g_2}{\partial x_1} \right| \leq \frac{e}{20} \quad \text{and} \quad \left| \frac{\partial g_2}{\partial x_2} \right| \leq \frac{e}{20}
\]

p. 448, line -3: it should be “A = A(x^*)” rather than “A = A(x)”.

p. 449, line 5: It should be “x ∈ S” rather than “x ∈ S”.

p. 451, line 16: In the displayed formula, there are two occurrences of “x^k” that should be “x^{(k)}”.
p. 480, first paragraph of 8.6.1: The first sentence should read

“In a homotopy method, one starts with a simple function $f(x), f : D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ such that every point with $f(x) = 0$ is known, then transforms the function into $g(x), g : D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ for which all points satisfying $g(x) = 0$ are desired.”

p. 485, first line of Exercise 27: It should be

“Suppose $f(x) = x^2 - 5x + 4, g(x) = (x - 2)(x + 2),”$

instead of

“Suppose $f(x) = (x - 2)(x + 2), g(x) = x^2 - 5x + 4$.”

Chapter 9

p. 517: The numbers on top of the nodes in Figure 9.6 are not correct.

p. 526, line 7: “Since $\varphi \in [0, 181]$” should be “Since $\varphi \in [0, 116]$”

p. 526, line 16: Remove the “=” from “$\varphi = \in [0, 41]$”

p. 526, line 17: It should be “Since $\varphi \in [0, 116]$” instead of “Since $\varphi \in [16, 106]$”

Chapter 10

pp. 543–544, and also Exercise 4 on page 567: The claim that Formula (10.21) is second order when $\alpha = \beta = 0$ is false. A counterexample is given in the Instructor’s Solution Manual for Exercise 4.

p. 544, last line: There is a misplaced comma. That is, “(10.25),” should be “(10.25),”.

p. 548, beginning of line 8: “since” should be “Since”.

p. 557, line 7: “we are obtain” should be “we obtain”.

pp. 567–568, Exercise 5: The reader is requested to prove

$$
\|Y - y\|_F^2 = \frac{1}{2} \int_0^1 (y'(x) - Y'(x))^2 dx \leq ch \max_{0 \leq x \leq 1} |y''(x)|.
$$

Instead, the reader should be requested to prove

$$
\|Y - y\|_F^2 = \frac{1}{2} \int_0^1 (y'(x) - Y'(x))^2 dx
$$

and

$$
\|Y - y\|_F \leq ch \max_{0 \leq x \leq 1} |y''(x)|.
$$
p. 569, Exercise 11: The reader should assume, as in Theorem 10.8 and Lemma 10.2, that \(|K| \leq M < 1\), and should also assume that \(K\) is symmetric, i.e. that \(K(s, t) = K(t, s)\). Additional clarification is given in the Instructor’s Solution Manual.

Appendix A

p. 571, second line of 1(c): An \(f(x)\) is missing at the end. That line should read:

\[
\min_{x \in [a,b]} f(x) \leq f(x_j) \leq \max_{x \in [a,b]} f(x)
\]

p. 574, first two lines of 7(b): There are two closing parentheses missing. That is, instead of

\[
\| (A + B)^{-1} - A^{-1} \| = \| (A + B)^{-1} (I - (A + B)A^{-1}) \|
\]

it should be

\[
\| (A + B)^{-1} - A^{-1} \| = \| (A + B)^{-1} (I - (A + B)A^{-1}) \|
\]