

Errata for
Classical and Modern Numerical Analysis:
Theory, Methods, and Practice
(for the first printing)

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Chapter 1

p. 2, in the proof of Theorem 1.2: There is an implicit assumption that $\int_a^b w(x)dx \neq 0$, and the case $\int_a^b w(x)dx = 0$ is not considered. To have the proof be complete, we need to observe that, if $\int_a^b w(x)dx = 0$, $w(x)$ must be identically equal to zero except on a set of measure zero. Thus, $\int_a^b f(x)w(x)dx = 0$, and Theorem 1.2 is also true in this special case.

p. 15, second line of Example 1.12: It should be $f\ell$ instead of $f\ell$.

p. 16, Theorem 1.8, its proof, and the definition of condition number: There is confusion between x^* and x , and the formula for the condition number should only depend on one of these. The following modifications will reduce the confusion:

$$\left| \frac{f(x) - f(x^*)}{f(x)} \right| \approx \left| \frac{x f'(x)}{f(x)} \right| \left| \frac{x - x^*}{x} \right|$$

PROOF The linear Taylor approximation of $f(x^*)$ about $f(x)$ for small values of $|x - x^*|$ is given by $f(x^*) \approx f(x) + f'(x)(x^* - x)$. Rearranging the terms immediately yields the result.

We now define the *condition number* of a function $f(x)$ as

$$\kappa_f(x) := \left| \frac{x f'(x)}{f(x)} \right|$$

p. 19, Table 1.1: Although only 23 bits are used for single precision and 52 bits are used for double precision, $t = 24$ for single precision and $t = 53$ for double precision, because the first digit is assumed to be 1.

p. 20, Table 1.2: ϵ_m for double precision should be $2^{-53} + 2^{-105} \approx 1.11 \times 10^{-16}$, and for single precision should be $2^{-24} + 2^{-45} \approx 5.96 \times 10^{-8}$.

p. 30, problem 2: What was defined was not exactly the traditional sinc function. The first part of Problem 2 should instead read:

Write down a polynomial $p(x)$ such that $|S(x) - p(x)| \leq 10^{-10}$ for $-0.2 \leq x \leq 0.2$, where

$$S(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

Note: $\text{sinc}(x) = S(\pi x) = \sin(\pi x)/(\pi x)$ is the “sinc” function (well-known in signal processing, etc.).

p. 66, (2.27): Instead of

$$\frac{\hat{x}_k - x}{x_k - \alpha} \rightarrow 0 \text{ as } k \rightarrow \infty,$$

it should be

$$\frac{\hat{x}_k - \alpha}{x_k - \alpha} \rightarrow 0 \text{ as } k \rightarrow \infty.$$

Chapter 3

p. 101, lines 1 to 4 of the proof: The sentence should read: “The proof rests on the result in linear algebra that any square matrix is similar to an upper triangular matrix, i.e., given any n by n matrix A , there exists a unitary matrix P such that $PAP^{-1} = \Lambda + U$, where Λ is diagonal and U is upper triangular with zeros on the diagonal.

p. 141, in (3.32): There is an extra “.”.

p. 143, line above (3.39): There should be no closing parentheses.

p. 154, line -8: It should be “ $i = 1, 2, \dots, n$ ”, rather than “ $1 = 1, 2, \dots, n$ ”.

p. 182, problem 23: Since an “operation” could mean a fused multiply-add, the phrasing of this question may cause some confusion. Also, it is unclear how to achieve n^3 multiplications. (See the analysis in the instructor’s answer guide.) Here is a suggested replacement for the problem:

Compute the number of multiplications it takes to compute the inverse of a matrix according to the note on page 110 as $cn^3 + \mathcal{O}(n^2)$. (That is, determine c .)

Hint: c will be smaller if you take advantage of the fact that the right-hand-sides of the systems you are trying to solve are the unit vectors e_j .

- p. 188, problem 55(b):** For consistency of notation with the rest of the text, it should be “find k ” and “ X_k ” rather than “find t ” and “ x_t ”.
- p. 188, problem 56(b):** Add the following sentence before the parenthetical note: “Here, $y_0 = b$ and the x_i are computed according to the Full Orthogonalization Method described on page 173.”
- p. 201, line 2:** There should not be a parenthesis after $(1, 0, 0)^T$.

Chapter 4

- p. 220, in (4.19):** It should be $f^{(2n)}(\xi)$ instead of $f^{2n}(\xi)$.
- p. 236, last row of Table 4.2:** This row is incorrect: K and M so $f = p + KM$ are not readily apparent for least squares, although bounds can be computed from the theory in this section.
- p. 237, line 13:** It should be “ $\sin([0, 0.05])$ ” instead of “ $\sin([0.0.05])$ ”.
- p. 237, lines 7, 10, 11, 14, 15, etc. and p. 238, line 2, etc.:** The last term in (4.26) should be $-\frac{1}{5040}x^7 \cos(\xi)$, rather than $-\frac{1}{5040}x^7 \sin(\xi)$. As a consequence, the numbers in the other indicated lines are incorrect. The page should be as follows:

$$\sin(x) \in x - \frac{x^3}{6} + \frac{x^5}{5!} - \frac{1}{5040}x^7 \cos(\xi) \quad \text{for some } \xi \in [-0.1, 0.1]. \quad (4.26)$$

We can replace $\cos(\xi)$ by an appropriate interval, say, by $[1 - x^2/2, 1]$, to get a pointwise estimate; for example,

$$\begin{aligned} \sin(0.05) &\in .05 - \frac{.05^3}{6} + \frac{.05^5}{120} - \frac{.05^7}{5040} [0.99875, 1] \\ &\subseteq [0.04997916927067, 0.04997916927068], \end{aligned}$$

where the above bounds are mathematically rigorous. Here, K was evaluated at the point x , but, $\cos(\xi)$ was replaced by the aforementioned bounds. Similarly,

$$\begin{aligned} \sin(-0.01) &\in (-.01) - \frac{(-.01)^3}{6} + \frac{(-.01)^5}{120} - \frac{(-.01)^7}{5040} [0.99995, 1] \\ &\subseteq [-0.00999983333417, -0.00999983333416]. \end{aligned}$$

Thus, since we know $\sin(x)$ is monotonic for $x \in [-0.01, 0.05]$,

$$[-0.00999983333417, 0.04997916927068]$$

represents a fairly sharp bound on the range $\{\sin(x) \mid x \in [-0.01, 0.05]\}$. Alternately, it may be more convenient in some contexts to evaluate K

and M over the entire interval, although this leads to a less sharp result. Using that technique, we would have

$$\begin{aligned} \sin(0.05) &\in .05 - \frac{.05^3}{6} + \frac{.05^5}{120} - \frac{0.05^7}{7!} [0.995, 1] \\ &\subseteq .05 - \frac{.05^3}{6} + \frac{.05^5}{120} + \\ &\quad [- - 0.15500992063493, -0.154234871031748 \times 10^{-12}] \\ &\subseteq [0.04997916927065, 0.04997916927071], \end{aligned}$$

and

$$\begin{aligned} \sin(-0.01) &\in (-.01) - \frac{(-.01)^3}{6} + \frac{(-.01)^5}{5!} - \frac{(-0.01)^7}{7!} \\ &\quad + \frac{[-0.1, 0.1]^*}{8!} [-0.1, 0.1] \\ &\subseteq (-.01) - \frac{(-.01)^3}{6} + \frac{(-.01)^5}{5!} - \frac{(-0.01)^7}{7!} [0.995, 1] \\ &\subseteq [-0.00999983333417, -0.00999983333416], \end{aligned}$$

thus obtaining (slightly less sharp) bounds

$$[-0.00999983333417, 0.04997916927071].$$

In general, substituting intervals into the polynomial approximation itself does not give sharp bounds on the range. For example,

$$\begin{aligned} \sin([-0.01, 0.05]) &\in ([-.01, .05]) - \frac{([-0.01, .05])^3}{6} + \frac{([-0.01, .05])^5}{120} \\ &\quad - \frac{([-0.01, .05])^7}{7!} [0.995, 1] \\ &\subseteq [-0.01002083333433, 0.05000016927084]. \end{aligned}$$

- pp. 238–249:** Instances of “ N ” here occur in contexts where, previously, “ n ” occurred (for example, in Definition 4.12 on page 238, and in previous sections). In these pages, such instances of “ N ” should be changed to “ n ”.
- p. 243:** There should be a comma after the $(x - x_{j-2})^3$ in the second to the last line of (4.29).
- p. 256:** In Remark 4.43, it should be “Euler’s formula” instead of “Euler’s identity”.
- p. 284, problem 1(d):** it should be “ $\varphi_3 \equiv t^3$ ”, rather than “ $\varphi_3 \equiv x^3$ ”.
- p. 284, problem 4:** It should be “Use the Gram–Schmidt” instead of ”“Use Gram-Schmidt”.

- p. 286, problem 13:** Since the $L_k(x)$ in this problem are the same as the $\ell_k(x)$ in (4.8) on page 212, L_k should be changed to ℓ_k , for consistency. Similarly, in problem 15 on the same page, l_i should be ℓ_i for consistency.
- p. 287, problem 18:** The polynomial should be of degree 3, so it should be referenced as $P_3(x)$, not as $P_2(x)$.
- p. 289, problem 32:** As printed, f is not uniquely defined at $x = \pi$. Replace “ $0 \leq x \leq \pi$ ” by “ $0 \leq x < \pi$ ”.
- p. 290, problem 38:** Part (a) is assumed. Thus, part (a) should be absent, and the problem should be to prove that φ is constant on $[0, 1)$. It is also helpful to assume some kind of continuity, such as $\lim_{x \rightarrow 0^+} \varphi(x) = \varphi(0)$. (Otherwise, φ could be set to an arbitrary value at a countable number of points, and still satisfy the recursion relation and orthogonality conditions with respect to Lebesgue measure.)

Chapter 5

- p. 321, problem 16:** It should be

$$x_{k+1} = -(A - 3I)^{-1}(A - 5I)^{-1}x_k,$$

instead of

$$x_{k+1} = -(A + 3I)^{-1}(A - 5I)^{-1}x_k,$$

- p. 326, formula on line 2:** Instead of:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} + \frac{h}{2}f''(\xi(x)).$$

it should be:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2}f''(\xi(x)).$$

- p. 331, line 17:** It should be “ $\cos(v_q)' - v_p'$ ”, rather than “ $\cos(v_q)' - v_p$ ”.
- p. 332, line -8:** It should be “ $\partial f/\partial x_i$ ” instead of “ df/dx_i ”.
- p. 332, lines -3 and -2:** It should be “ $\partial f/\partial x_1$ ” and “ $\partial f/\partial x_2$ ” instead of “ df/dx_1 ” and “ df/dx_2 ”.

Chapter 6

- p. 346, first line of Corollary 6.1:** It should be “ $\{p_i\}_{i \geq 0}$ ” instead of “ $\{p\}_{i \geq 0}$ ”.
- p. 349, line 2:** It should be “ $z_j, 0 \leq j \leq m$ ” instead of “ $z_j, 0 \leq j \leq n$ ”.

p. 349, Table 6.1: The indexing on the α 's and z 's should go from 0 to m , rather than from 1 to $m + 1$, to be consistent with the rest of the text.

p. 349, line 2 of Lemma 6.1: It should be “of at most degree $2m+1$ ” instead of “of a most degree $2m + 1$ ”.

p. 350, line -7: It should be “ $\frac{f^{(2m+2)}(\xi)}{(2m+2)!}$ ” instead of “ $\frac{f^{2m+2}(\xi)}{(2m+2)!}$ ”.

p. 352, line -5: It should be “ $\prod_{\substack{i=0 \\ i \neq j}}^m (x_j - x_i)^2$ ” instead of “ $\prod_{\substack{i=0 \\ i \neq j}}^n (x_j - x_i)^2$ ”.

p. 367, last line: It should be “integrals over infinite intervals” instead of “integrals over infinite integrals”.

p. 377, problem 7: The authors do not know a general closed-form formula for the n -th component in (6.14). A suggested rewriting of problem 7 is:

Compute the fourth and fifth components in (6.14), assuming order 4 Taylor arithmetic is to be used. Can a general formula be derived? Can a recursive routine be devised to compute the n -th term, without explicitly computing it by hand first?

Instructors can consult the solutions manual for an answer to this modified question.

p. 377, problem 8: There is a similar difficulty with this problem as with problem 7. It is suggested that the problem be replaced by:

Compute the components of the degree-4 Taylor object for $(u_{\nabla})^n$.

Project: Write a program that recursively computes a general coefficient for a degree N Taylor object for $(u_{\nabla})^n$.

p. 378, problem 15: This problem is erroneously stated. As a counterexample to part (a), take $f(x) = x^{10}$, and $a = 10$. Then, $f(-a) + f(a) = 2 \cdot 10^{10}$, while $\int_{-1}^1 f(x) dx = 2/11$, and the error bound is only $(10^3 - 10^2 + \frac{1}{3}) \cdot 90$.

p. 379, line 1: For consistency with the rest of the text, “Trapezoid rule” should be “trapezoidal rule”.

Chapter 7

p. 383, line 10: Instead of “on page 482 in Section 8.1,” it should be “on page 482 in Section 8.7.”

p. 419, paragraph beginning on line 6: The second sentence is erroneous. It should read

A spring is “stiff” if its damping constant is large; in such a mechanical system, motions of the spring will damp out fast relative to the time scale on which we are studying the system.

instead of

A spring is “stiff” if its spring constant is large; in such a mechanical system, the spring will cause motions of the system that are fast relative to the time scale on which we are studying the system.

p. 433, line 4 of problem 16: It should be “determine β ” instead of “determine β ”.

p. 435, problem 24: (i) Method (i) Should read

$$y_{j+2} - 2y_{j+1} + y_j = 2h (f(t_{j+1}, y_{j+1}) - f(t_j, y_j)).$$

(iii) Method (iii) should read

$$y_{j+1} - y_j = hf(t_{j+1}, y_{j+1}).$$

Finally, the precise problem being solved is irrelevant to consistency and stability of the method, so the first sentence of the problem should be deleted.

Chapter 8

p. 443, line -13: It should be “ $G(x(k))$ ” rather than “ $G(x^k)$ ”.

p. 445, last line: It should be “ $\Phi'(s)$ ” rather than “ $\Phi'_j(s)$ ”.

p. 446, lines 6 and 8: the derivative signs in the Leibnitz notation should be partial derivative signs. That is, lines 6, 7, and 8 should read:

$$G'(x) = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}x_2 \sin(x_1x_2) & \frac{1}{3}x_1 \sin(x_1x_2) \\ \frac{x_2}{20}e^{-x_1x_2} & \frac{x_1}{20}e^{-x_1x_2} \end{pmatrix}$$

and

$$\left| \frac{\partial g_1}{\partial x_1} \right| \leq \frac{1}{3}, \quad \left| \frac{\partial g_1}{\partial x_2} \right| \leq \frac{1}{3}, \quad \left| \frac{\partial g_2}{\partial x_1} \right| \leq \frac{e}{20}, \quad \text{and} \quad \left| \frac{\partial g_2}{\partial x_2} \right| \leq \frac{e}{20}$$

p. 448, line -3: it should be “ $A = A(x^*)$ ” rather than “ $A = A(x)$ ”.

p. 449, line 5: It should be “ $x \in \bar{S}$ ” rather than “ $\bar{x} \in \bar{S}$ ”.

p. 451, line 16: In the displayed formula, there are two occurrences of “ x^k ” that should be “ $x^{(k)}$ ”.

p. 480, first paragraph of 8.6.1: The first sentence should read

“In a homotopy method, one starts with a simple function $f(x)$, $f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that every point with $f(x) = 0$ is known, then transforms the function into $g(x)$, $g : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$ for which all points satisfying $g(x) = 0$ are desired. ”

p. 485, first line of Exercise 27: It should be

“Suppose $f(x) = x^2 - 5x + 4$, $g(x) = (x - 2)(x + 2)$,”

instead of

“Suppose $f(x) = (x - 2)(x + 2)$, $g(x) = x^2 - 5x + 4$ ”.

Chapter 9

p. 517: The numbers on top of the nodes in Figure 9.6 are not correct.

p. 526, line 7: “Since $\bar{\varphi} \in [0, 181]$ ” should be “Since $\bar{\varphi} \in [0, 116]$ ”

p. 526, line 16: Remove the “=” from “ $\bar{\varphi} = \in [0, 41]$ ”

p. 526, line 17: It should be “Since $\bar{\varphi} \in [0, 116]$ ” instead of “Since $\bar{\varphi} \in [16, 106]$ ”

Chapter 10

pp. 543–544, and also Exercise 4 on page 567: The claim that Formula (10.21) is second order when $\alpha = \beta = 0$ is false. A counterexample is given in the Instructor’s Solution Manual for Exercise 4.

p. 544, last line: There is a misplaced comma. That is, “(10.25,)” should be “(10.25),”.

p. 548, beginning of line 8: “since” should be “Since”.

p. 557, line 7: “we are obtain” should be “we obtain”.

pp. 567–568, Exercise 5: The reader is requested to prove

$$\|Y - y\|_F^2 = \frac{1}{2} \int_0^1 (y'(x) - Y'(x))^2 dx \leq ch \max_{0 \leq x \leq 1} |y''(x)|.$$

Instead, the reader should be requested to prove

$$\|Y - y\|_F^2 = \frac{1}{2} \int_0^1 (y'(x) - Y'(x))^2 dx$$

and

$$\|Y - y\|_F \leq ch \max_{0 \leq x \leq 1} |y''(x)|.$$

- p. 569, Exercise 11:** The reader should assume, as in Theorem 10.8 and Lemma 10.2, that $|K| \leq M < 1$, and should also assume that K is symmetric, i.e. that $K(s, t) = K(t, s)$. Additional clarification is given in the Instructor's Solution Manual.

Appendix A

- p. 571, second line of 1(c):** An $f(x)$ is missing at the end. That line should read:

$$\min_{x \in [a, b]} f(x) \leq f(x_j) \leq \max_{x \in [a, b]} f(x)$$

- p. 574, first two lines of 7(b):** There are two closing parentheses missing. That is, instead of

$$\begin{aligned} \|(A + B)^{-1} - A^{-1}\| &= \|(A + B)^{-1}(I - (A + B)A^{-1})\| \\ &= \|(A + B)^{-1}(I - AA^{-1} - BA^{-1})\| \end{aligned}$$

it should be

$$\begin{aligned} \|(A + B)^{-1} - A^{-1}\| &= \|(A + B)^{-1} (I - (A + B)A^{-1}) \| \\ &= \|(A + B)^{-1} (I - AA^{-1} - BA^{-1}) \| \end{aligned}$$