Index

A(0) stability, 425
A-stable
  method for an IVP, 423
absolute error, 14
absolute stability
  methods for a scalar equation, 400
  methods for systems, 421
  predictor-corrector method, 418
accuracy, order of
  multistep method, 407
Adams methods, 406, 417
Adams–Bashforth methods, 406
Adams–Moulton methods, 406
adaptive quadrature, 374
Aitken acceleration, 65
approximation
  best, 192
  least squares, 198
  minimax, 230
Arnoldi’s Algorithm, 171
asymptotic rate of convergence, 161
automatic differentiation, 328
  forward mode, 328
  reverse mode, 330
automatically verified, 519
B-spline, 243
  basis, 244
back substitution, 106, 115
back solving, 106
backward error analysis, 126
Banach space, 197
basic feasible point
  of a linear program, 506
basis
  B-spline, 244
  collocating, 212
Haar, 274
hat function, 239
Lagrange, 212
linear programming, 506
  of a vector space, 89
basis functions, 140
Bernoulli number, 358
Bernoulli numbers, 355
Bernstein polynomials, 206
boundary value problem, 535
bound constraints, 487
boundary value problems
  finite difference methods, 540
  shooting method, 536
bounded variation, 250
branching step, 523
branch and bound algorithm, 74, 374, 523
  clustering effect, 530
  for nonlinear systems of equations, 481
branching step, 523
Brouwer fixed point theorem, 471
Broyden update, 474
Broyden’s method, 475

599
Index

Broyden–Fletcher–Goldfarb–Shanno update, 496
Cauchy polygon method, 385
Cauchy sequence, 40, 197
Cauchy–Schwarz inequality, 90
central difference formula, 324
Central Limit Theorem, 369
c centroid
    of a simplex, 498
chapeau functions, 547
characteristic equation, 87
characteristic polynomial, 291
Chebyshev norm, 191
Chebyshev polynomials, 217, 224
Chebyshev’s Equi-Oscillation Theorem, 234
Cholesky factorization, 117
chop, 11
clamped spline interpolant, 246
clustering effect, branch and bound
    algorithm, 530
code list, 329
collocating basis, 212, 240
companion matrix, 74, 170
compatible matrix and vector norms, 96
complete search algorithms, 519
complete space, 40
complex Fourier series, 250
complex reflector, 135
composite integration, 334
condition
    ill, 122
    number
        generalized, 180
        of a function, 16
        of a matrix, 123
        perfect, 124
        Wilkinson polynomial, 73
consistency
    of a k-step method, 409
    of a method for solving an IVP, 391
    consistently ordered matrix, 158
constraint programming, 529
constraint propagation, 527
constraint satisfaction problem, 488
continuation method, 481
continuity
    modulus of, 230
contraction, 40
Contraction Mapping Theorem, 442
    in one variable, 40
convergence
    global, 454
    iterative method for linear systems, 142
    linear, 7
    local, 454
    of a sequence of matrices, 102
    of a sequence of vectors, 92
    of the SOR method, 147
    order of, 7
    quadratic, 7
    rate
        asymptotic, 161
        semilocal, 454
        superlinear, 8
convex
    function, 455
    program, 488, 515
    set, 444
    strictly, 475
correct rounding, 23
critical point
    of a constrained optimization problem, 502
    of an unconstrained optimization problem, 492
cubic spline, 243
cyclic matrix, 158
Daubechies scaling function, 278
Davidon–Fletcher–Powell update, 496
defective matrix, 292
deflation
    eigenvalues of matrices, 305
    roots of polynomials, 72
dependency, interval, 27
derivative tensor, 441
Descartes’ rule of signs, 70
DFP update, 496
diagonally dominant, 150
    irreducibly, 150
    strictly, 111, 150
differentiation
    automatic, 328
dilates, 270
dimension
    of a vector space, 89
Dirichlet problem, 535
discretization error, 387
distance
    in a normed space, 92
divided difference
    $k$-th order, 213
    first order, 63, 213
    Newton’s backward formula, 215
    Newton’s formula, 214
    second order, 63
domain
    frequency, 258
    time, 258
Doolittle algorithm, 185
double QR algorithm, 312
dual problem, 502
dual variables, 502
dynamic programming, 515
eigenvalue, 86, 291
    simple, 297
eigenvector, 86, 291
elementary row operations
    for linear systems, 88
    in the simplex method, 508
equi-oscillation property
    minimax, 234
equilibration, row, 125
equivalent norms, 93
error
    absolute, 14
    backward analysis, 126
bound for a single-step method
    for IVP, 393
forward analysis, 126
local truncation
    for a single-step method for
    IVP’s, 391
method, 9
relative, 14
roundoff, 9
roundout, 25
truncation, 9
Euclidean norm, 90
Euler’s method, 385
Euler–Maclaurin formula, 355
Euler–Trapezoidal Method, 417
excess width, 27
Exchange Method of Remez, 235
explicit method
    for solving IVP’s, 406
explicit single-step method, 389
extended precision, 389
extended real numbers, 24
Fast Fourier Transform, 256
fathomed, 526
feasible point, 507
feasible set, 488
feasible solution, 507
FFT, 256
Fibonacci sequence, 62
filtering, 258
finite difference methods, 540
finite element method, 547
fixed point, 39, 442
    iteration method, 39
floating point numbers, 10
forward difference formula, 215, 323
forward error analysis, 126
forward mode, automatic differentiation, 328
Fourier series, 204
    complex, 250
Fréchet derivative, 440
Fredholm integral equation
    of the second kind, 554, 559
existence and uniqueness, 560
numerical solution of, 562
frequency domain, 258
Fritz John conditions, 503
Frobenius norm, 96
full pivoting
  Gaussian elimination, 114
full rank matrix, 86
Fundamental theorem of algebra, 69
fundamental theorem of interval arithmetic, 26
GA’s, 520
Galerkin method, 545, 549
  for Fredholm integral equations of the second kind, 563
  functional analysis setting, 549
Gauss–Legendre quadrature, 348
Gauss–Seidel method, 143
Gauss–Seidel–Newton method, 461
different from Newton–Gauss–Seidel Method, 462
Gaussian elimination, 105
  full pivoting, 114
  partial pivoting, 114
Gaussian quadrature, 343
  2-point, 344
  definition, 344
  error term, 349
Gauss–Legendre rules, 348
generalized condition number, 180
Generalized Rolle’s Theorem, 216
genetic algorithms, 520
Gerschgorin’s Circle Theorem, 293
  for Hermitian matrices, 296
Givens rotation, 137, 316
global convergence, 454
global minimizer, 489
global optimization, 489, 518
global truncation error, 387
golden mean, 491
golden section search, 490
Gram matrix, 198
Gram–Schmidt process, 138, 201
  modified, 139, 171
graph of a matrix, 157
Haar basis, 274
Halton sequence, 371
harmonic analysis, 258
hat functions, 239, 547
Hermite interpolating polynomial, 220
Hermite interpolation, 220
Hermite polynomials, 224
Hermitian matrix, 87, 295
Hessenberg matrix, 307
Hessian matrix, 492
heuristic, 497
Hilbert matrix, 125
Hilbert space, 197
homotopy method, 480
  predictor-corrector method, 481
Horner’s method, 71
Householder transformation, 133, 305
HUGE, 20
identity matrix, 86
IEEE arithmetic, 19
ill-conditioned, 122
implicit method
  for solving IVP’s, 406
  implicit Simpson’s method, 405
  implicit trapezoid method, 434
  improper integrals, 365
  improved Euler method, 397
  independent, linearly, 89
  infeasible linear program, 513
  infimum, 194
  infinite integrals, 365
  initial value problem, 381
  inner product, 91
  real space, 195
integration, 333
  composite, 334
  infinite, 365
  midpoint rule, 341
  Monte Carlo, 369
  multiple, 364
  singular, 365
interior point methods, 507
interpolating polynomial
    Hermite, 220
    Lagrange form, 210, 212, 338
    Newton form, 214
interpolation
    Hermite, 220
interval arithmetic
    fundamental theorem of, 26
    operational definitions, 24
interval dependency, 27
interval extension
    first order, 28
    mean value, 33
    multivariate mean value, 467
    second order, 28
interval Gauss–Seidel method
    nonlinear, 465
interval Newton
    operator, 464
    univariate, 54
interval Newton method
    multivariate, 463
    quadratic convergence of, 57
    univariate, 55
inverse
    of a matrix, 86
    inverse midpoint matrix, 154
    inverse power method, 303
invertible matrix, 86
irreducible, 150
irreducible graph, 157
irreducibly diagonally dominant, 150
iterative refinement, 127
IVP, 381
Jacobi diagonalization, 315
Jacobi method, 143
    for computing eigenvalues, 315
    nonlinear, 461
Jacobi rotation, 316
Jacobi–Newton Method, 461
Jacobian matrix, 440
Kantorovich Theorem, 454
Kantorovich theorem, 51
Karmarkar’s algorithm, 507
    kernel
    of an integral equation, 554
Krawczyk
    operator, 471
    Krawczyk method
    multivariate, 470
    univariate, 81
Kronecker delta function, 92
Krylov subspace, 169
Kuhn–Tucker equations, 501
Kuhn–Tucker point, 502
L-2 norm, 192
Lagrange
    basis, 210, 212
    polynomial interpolation, 210, 212, 324, 338
Laguerre polynomials, 224
Lanczos Algorithm, 172
Lax–Milgram lemma, 550
least squares
    approximation, 140, 198, 279
    general least squares problem, 222
    norm, 192
    left singular vector, 175
Legendre polynomials, 203, 223
Leibnitz rule, 264
Lemaréchal’s technique, 532
line search, 490
    golden section, 490
linear convergence, 7
linear least squares problem, 279
linear program, 487, 488
    infeasible, 513
    standard form, 504
    unbounded, 513
linear programming
    interior point methods, 507
    linear relaxation, 527
    linearly independent, 89
    Lipschitz condition, 40, 382, 555
    Lipschitz matrix, 463
    local convergence, 454
    local minimizer, 489
local truncation error
  finite difference methods for boundary value problems, 542
  of a method for solving IVP’s, 391
low discrepancy sequence, 371
LU
decomposition, 109
factorization, 109
machine constants, 20
machine epsilon, 20
mag, 156
magnitude (of an interval), 156
mantissa, 10
matrix norm, 95
  compatible, 96
  Frobenius, 96
  induced, 97
  natural, 97
max norm, 191
mean value interval extension, 33
  multivariate, 467
mean value theorem
  for integrals, 2
  multivariate, 441
  univariate, 3
method error, 9
method of bisection, 36
midpoint method
  for solution of initial value problems, 390
midpoint rule
  for quadrature, 341, 342, 349
mig, 156
magnitude (of an interval), 156
minimax approximation, 230
minimax equi-oscillation property, 234
Miranda’s Theorem, 468
mixed boundary conditions, 544
modified Euler method, 397
modified Gram–Schmidt procedure, 139
modified Gram–Schmidt process, 171
modulus of continuity, 230
monic polynomial, 218
Monte Carlo integration, 369
Monte Carlo method
  quasi, 371
Moore–Penrose pseudo-inverse, 176
Muller’s method, 63
multi-stage decision processes, 515
multiple integrals, 364
multistep method, 405
multivariate interval Newton operator, 464
multivariate mean value theorem, 441
NaN, 20
natural or induced matrix norm, 97
natural spline, 246
near minimax approximation, 235
Nelder–Mead simplex method, 497
Newton’s backward difference formula, 215
Newton’s divided difference formula, 214
Newton’s forward difference formula, 215
Newton’s method
  convergence of, 50
  multivariate, 447
  local convergence of, 450
  univariate, 49
Newton–Cotes formulas, 336
  closed, 336
  open, 336
Newton–Gauss–Seidel Method, 460
  different from Method, 462
Newton–Gauss–Seidel method
  different from Gauss–Seidel–Newton
  Method, 462
Newton–Kantorovich Theorem, 454
Newton–SOR iteration, 1-step, 460
node
  of a graph, 157
nondefective matrix, 292
nonlinear interval Gauss–Seidel method, 465
nonlinear Jacobi method, 461
nonlinear program, 487
nonsingular matrix, 86
nonsmooth optimization, 532
norm
\[ L^2, \] 192
Chebyshev, 191
equivalent, 93
Euclidean, 90
important ones on \( \mathbb{C}^n \), 90
least squares, 192
matrix, 95
compatible, 96
Frobenius, 96
induced, 97
natural, 97
max, 191
of a vector space, 88
uniform, 191
normal distribution
standard, 214
normal equations, 140, 198
normed vector space, 88
not a number, 20
NP-complete problem, 488
objective function, 487
operator overloading, 333
optimization
constrained, 487
convex, 488
unconstrained, 487
optimizer, 488
optimizing point, 488
optimum, 488
order
of a single-step method for solving an IVP, 391
of convergence, 7
order of accuracy
multistep method, 407
predictor-corrector method, 417
origin shift, 308
orthogonal complement, 199
orthogonal decomposition, 132
orthogonal polynomials, 222, 345
orthogonal projection, 200
Ostrowski, a lemma of, 449
outward rounding, 25
overflow, 20
overloading, operator, 333
overrelaxation factor, 145
Padé approximant, 260
Padé methods
for IVP's, 424
partial pivoting
Gaussian elimination, 114
pattern search algorithms, 497
perfectly conditioned, 124
perpendicular (from a point to a set), 200
Perron–Frobenius theorem, 150
plane rotation, 137, 316
Poincaré's inequality, 553
polynomial time algorithm, 488
positive
definite, 87
semi-definite, 87
preconditioning, 154
predictor-corrector method
for a homotopy method, 481
for systems of ordinary differential equations, 416
primal variables, 502
product formula, 364
projection operator, 199
property A, 158
pseudo-inverse, 176
QR
decomposition, 132
factorization, 132
method, 307
convergence of, 309
double, 312
with origin shifts, 308
quadratic convergence, 7
quadratic programming, 514
quadrature, 333
composite, 334
Gaussian, 343
to zero, 19
to, 19
up, 19
rounding modes, 19
roundoff error, 9
in Gaussian elimination, 126
roundout error, 25
row equilibration, 125
Runge’s function, 219, 284
Runge–Kutta method, 396
fourth order classic, 398
R-stage, 397
stability of, 398

quasi-Monte Carlo method, 371
quasi-Newton
Davidon–Fletcher–Powell update, 496
equation, 473
method, 473
quasi-random sequence, 371

R-stage Runge–Kutta method, 397
rank
of a matrix, 86
rational approximation, 259
Rayleigh quotient, 300
real inner product space, 195
rectangle rule, 364
reducible, 150
reflector
complex, 135
relative error, 14
relaxation, 524
linear, 527
relaxation direction, 164
Remez Algorithm, 235
residual vector, 127, 163
Richardson extrapolation, 354
right singular vector, 175
Rolle’s Theorem, 216
Generalized, 216
Romberg integration, 355
round, 11
down, 19
to nearest, 19

Schur decomposition, 101, 292

Schwarz inequality, 90

search tree, 526
secant equation, 473
secant method, 59
convergence of, 60
secant update, 479
semi-definite, 87
semilocal convergence, 454
sequence
Cauchy, 197
Sherman–Morrison formula, 476
shooting method, 536
significant digits, 17
similarity transformation, 292
simple eigenvalue, 297
simplex method
of linear programming, 507
of Nelder and Mead, 497
Simpson’s method
for IVP, 405
Simpson’s rule, 342
simultaneous iteration, 318
single use expression, 27
single-step methods, 389
singular integrals, 365
singular vector
left, 175
right, 175
slope matrix, 464
smoothing polynomials, 281
smoothness, 373
solution set, 131
SOR
  matrix, 146
  method, 145
SOR method
  block, 161
  convergence of, 147
span, 89
sparse matrix, 157
spectral radius, 87, 291
spectrum, 291
spline
  B-, 243
  clamped, 246
  cubic, 243
  natural, 246
stability
  A(0), 425
  predictor-corrector method, 418
  Runge–Kutta methods, 398
standard form, linear program, 504
standard normal distribution, 214
steepest descent method, 493
Steffensen’s method, 68
Stein’s Theorem, 147
Stein–Rosenberg theorem, 149
stiff
  system of ODE’s, 419, 422
strictly convex, 475
strongly connected directed graph, 157
subdistributivity, 25
subspace
  Krylov, 169
  of a vector space, 89
subspace iteration, 318
successive overrelaxation, 145
successive relaxation method, 143
SUE, 27
superlinear convergence, 8
symmetric matrix, 87
symmetric quadrature rule, 363
synthetic division, 71
tape, 329
taylor polynomial
  approximation by, 209
  multivariate, 442
taylor series methods
  for solving IVP’s, 395
taylor’s theorem, 2
tensor
  derivative, 441
time domain, 258
TINY, 20
total step method, 143
tractable problems, 488
trapezoid method
  implicit, 434
  trapezoidal rule, 340, 342
triangle inequality, 89, 90
  for 2-norm, 91
  when strict inequality, 485
triangular
  decomposition, 109
  factorization, 109
truncation error, 9
global, 387
Tschebycheff polynomials, 217
two-cyclic matrix, 158
two-point compactification, 24
unbounded linear program, 513
unconstrained optimization, 487
underflow, 20
underrelaxation factor, 145
uniform norm, 191
unitary matrix, 124
update
  quasi-Newton, 495
Vandermonde matrix, 211, 338
variation, bounded, 250
vector space
  normed, 88
Volterra integral equation
  of the second kind, 554
  numerical solution of, 557
wavelet, 273
weak formulation, 544
Weierstrass approximation theorem, 205
Wilkinson polynomial, 73, 82
Wilkinson Prize, 82